NEAR – FIELD COHERENT EFFECTS AT THERMAL MICROWAVE RADIATION RECEIVING ON COUPLED LINEAR WIRE ANTENNAS

Y. N. Barabanenkov[1], M. Yu. Barabanenkov[2], V. A. Cherepenin[1]

[1] V.A. Kotelnikov Institute of Radioengineering and Electronics of RAS, Moscow

[2] Institute of Microelectronics Technology of RAS, Chernogolovka

Received December 4, 2011

БЛИЖНЕПОЛЕВЫЕ КОГЕРЕНТНЫЕ ЭФФЕКТЫ ПРИ ПРИЕМЕ ТЕПЛОВОГО МИКРОВОЛНОВОГО ИЗЛУЧЕНИЯ ВЗАИМОДЕЙСТВУЮЩИМИ ЛИНЕЙНЫМИ ПРОВОЛОЧНЫМИ АНТЕННАМИ

Ю. Н. Барабаненков[1], М. Ю. Барабаненков[2], В. А. Черепенин[1]

[1] Институт радиотехники и электроники им. В.А. Котельникова РАН, Москва
[2] Институт проблем технологии микроэлектроники и особочистых материалов РАН, Черноголовка

Abstract. We apply to study coupled receiving antennas a theory of electromagnetic wave multiple scattering by ensemble of dielectric and conductive bodies, with describing the excited currents inside bodies in terms of electric field tensor T-scattering operator. A system of equations for currents on surfaces of coupled perfectly conductive receiving antennas is written with the aid of a single antenna surface T-scattering operator. This system is resolved for the case of coupled linear wire receiving antennas in the form of thin vibrator-dipoles when asymptotic method of "big logarithm" leads to separable wire T-scattering operator of single tuned vibrator-dipole. The separability simplifies analytic evaluating the local total currents on two (and more) coupled receiving antennas and get a dimensionless coupling factor. Our final aim consists in using the obtained analytic solution to study near field coherent effects caused by thermal microwave radiation incident electric field distribution along single or two coupled receiving vibrator-dipole antennas placed at a heated biological object boundary surface and tuned to half wavelength in the object. In the case of equilibrium thermal radiation we meet a generalized Nyquist formula for currents' fluctuations excited on coupled receiving vibrator-dipole antennas, with accounting the auto-correlation and cross-correlation functions of random electric field inside each antenna and on both antennas, respectively. In the case of local volume change of the biological object temperature distribution we reveal in the model framework of random electric dipole source inside object absorption skin slab area the interference-extreme properties for fluctuations of currents excited along antennas depending on relative positions of antennas and random electric dipole source. The reveled extreme properties are used as base to reconstruct the random electric dipole position via scanning the single antenna or two coupled ones along biological object boundary surface.

Key words: receiving coupled antennas, excited currents, T- scattering operator, thin vibrator antennas, biological object, temperature local variation, thermal radiation, interference receiving fields, scanning local random source.

Аннотация. Мы применяем теорию многократного рассеяния электромагнитных волн на ансамбле диэлектрических и проводящих тел для изучения взаимодействующих принимающих антенн, описывая возбужденные внутри антенн токи с помощью тензорного Т –оператора рассеяния электрического поля. Система уравнений для токов на поверхностях взаимодействующих металлических принимающих антенн записывается с помощью поверхностного Т- оператора рассеяния изолированной антенны. Записанная система решается для случая взаимодействующих линейных проволочных принимающих антенн в форме тонких вибраторов – диполей асимптотическим методом " большого логарифма ", приводящим к сепарабельному линейному Т- оператору рассеяния изолированного настроенного вибратора-диполя. Сепарабельность упрощает аналитическое вычисление локальных токов на двух (и более) взаимодействующих антеннах и безразмерного параметра взаимодействия антенн. Наша окончательна цель состоит в использовании полученного аналитического решения к изучению ближнеполевых когерентных эффектов, обусловленных распределением падающего электрического поля теплового микроволнового излучения вдоль изолированного или двух взаимодействующих принимающих вибраторов-диполей, расположенных около граничной поверхности нагретого биологического объекта и настроенных на половину длины волны в объекте. При равновесном тепловом излучении мы приходим к обобщенной формуле Найквиста для флуктуаций токов, наведенных на взаимодействующих принимающих вибраторах-диполях, с учетом авто-корреляционной и кросс-корреляционной функций случайного электрического поля на каждом или обоих вибраторах-диполях, соответственно. Рассматривая локальное объемное изменение распределения температуры биологического объекта, мы обнаруживаем в рамках модели случайного электрического дипольного источника внутри поглощающего скин-слоя объекта интерференционно-экстремальные свойства возбуждаемых вдоль принимающих вибраторов – диполей флуктуационных токов в зависимости от относительного расположения вибраторов-диполей и случайного электрического дипольного источника теплового излучения. Обнаруженные экстремальные свойства используются как основа для восстановления расположения случайного электрического дипольного источника путем сканирования изолированным или двумя взаимодействующими принимающими вибраторами-диполями вдоль граничной поверхности биологического объекта.

Ключевые слова: принимающие взаимодействующие антенны, возбужденные токи, T- оператор рассеяния, тонкая вибраторная антенна, биологический объект, локальная вариация температуры, тепловое излучение, интерференция принимаемых полей, сканирование локального случайного источника.

Introduction

Measuring method of the temperature spatial distribution inside of a biological tissue object by recording its own thermal radiation in the microwave range is well known at the present time [1]. During the last twenty years, there has been realized that a contact antenna [2-4] or system of contact antennas [5] are situated in the area of a heated object thermal near fields, which have basically the form of evanescent electromagnetic waves exponentially decaying in perpendicular to the object boundary surface direction accordingly with Rytov's prediction [6].

Note, mean size D of a contact antenna aperture is rationally to be chosen in accordance with [2-5] as being much smaller than radiation wavelength λ in free space and of the order of the radiation wavelength λ_1 inside a biological object, i.e., $\lambda_1 \le D << \lambda$. Such inequalities suppose the real part ε' of biological tissue complex dielectric permittivity $\varepsilon = \varepsilon' + i\varepsilon''$ in microwave range to be substantially greater the dielectric permittivity ε_0 of free space that is really the case for some tissues. For example, the dielectric permittivity of human head brain may be taken at frequency f = 780 MHz to be $\varepsilon = 30 + i8.5$ [5] that gives λ = 39 cm, λ_1 = 7 cm and absorption skin depth d_{sk} = $1/2k_1''$ = 4 cm where k_1 = $k_0\sqrt{\varepsilon}$ = $k_1' + ik_1''$ is the complex wave number inside brain medium, with k_0 = $2\pi/\lambda$ and k_1' = $2\pi/\lambda_1$. Under above inequalities putting on mean size aperture, the contact antenna receives strong decaying evanescent waves of thermal radiation in free space, which correspond to weak decaying evanescent waves inside biological object.

The wave theory [6,7,8] of an absorbing body thermal electromagnetic radiation having been published , Levin and Rytov [9] and Rytov *et al* [10] have considered some problems on current excitation in metallic antennas by thermal radiation fields, including the case of thin metallic antennas (linear wire antennas). To considered problems are related: (i) antenna excitation in equilibrium thermal radiation field; (ii) current excitation in antenna by thermal radiation of distant bodies; (iii) current excitation in coupled antennas, with phenomenological accounting of coupling effect by antenna mutual impedance.

The aim of our presented work is to consider the current excitation in coupled receiving antennas by incident electromagnetic field, with evaluating the coupling effect via analytic solution to system of equations for currents' distribution on coupled linear wire tuned antennas. Especially we intend to study the current excitation in such antennas by thermal radiation of small body placed in near field zone of antennas, as well as by equilibrium thermal radiation. Actually our work consists in the following.

Because electromagnetic mutual interaction (coupling) the antennas is included in theory of wave multiple scattering by ensemble of bodies [11], we write the volume current density inside a body in terms of the body electric field tensor T-scattering operator [12-17] and then derive the general system of equations for volume current densities inside all bodies of ensemble ones with the aid of Watson composition rule [12, 15] for scattering operators. A specific property of the derived system of equations for currents' densities inside bodies consists in that the system is written in terms of single body T-scattering operator. In particular case of antennas placed at a heated biological object boundary surface the derived general system of equations for volume currents' densities inside antennas takes into account also the coupling effects between antennas and object boundary surface (inhomogeneous object) that we do neglect in our concrete calculations. What is more, bearing in mind that mean size D of contact antennas conforms to wavelength λ_1 inside a biological object, we think them as were placed in the object boundary subsurface, with thickness being not more or order the absorption skin depth d_{sk}. On the way of such simplifications the mentioned general system of equations is transformed to a reduced system of equations for surface currents' densities on surfaces of coupled perfectly conducting receiving antennas written with the aid of a single antenna surface T-scattering operator. The reduced system supposes all coupled antennas to be placed in an effective unbounded medium, with wave number being equal to wave number k_1 inside the biological object under study. The next our step consists in passing to linear wire antennas, with mean diametrical size a of each wire being very small compared to wire length D and wavelength λ_1, $a << D$; $a << \lambda_1$. Passing to linear wire replaces the above single antenna surface T-scattering operator to single wire T-scattering operator that is one-dimensional kernel, which expresses the total current in a cross section of the wire through excited electric field component along wire. According to Levin and Rytov [9], the single wire T-scattering operator of tuned linear wire antenna, $D = n\lambda_1/2$ (n is integer number), takes a simple separable form in asymptotic limit of "big logarithm" when parameter $\eta = 1/(2\ln k_1'a)$ has a small value. Theory of this asymptotic limit was elaborated by Leontovich and Levin [18] after Hallen paper [19] for analytic solution to Pocklington integral equation [20] that describes the current distribution along linear wire antenna depending on excited electric field component along the wire. We use the separable form of the single wire scattering operator to resolve the system of equations for currents' distributions along two coupled tuned linear wire antennas in terms of excited electric field components along both wires and their coupling factor, which is defined as ratio of antenna mutual impedance to self-impedance of a single antenna and evaluated directly. The mutual impedance of two wire antennas was studied earlier [21, 22] for transmitting antennas with the aid of more complicated method; convenient formulas for mutual impedance of antennas placed on big distant between them are considered in paper [23]. The obtained currents' distributions along coupled tuned linear wire antennas are applied to the case when exciting electric field is caused by thermal radiation of a biological object, with temperature being equal to sum of homogeneous component Θ_0 and temperature local volume spatial variation $\delta\Theta(\vec{r})$. The homogeneous component of the object temperature Θ_0 creates an equilibrium thermal radiation that is characterized by standard form [10] of electric field spatial correlation function as electric field tensor Green function imaginary part, with a scalar factor being in front of it. A local volume variation of the object temperature $\delta\Theta(\vec{r})$ gives rise to the thermal radiation from corresponding local random electric current density variation $\delta\vec{j}^{\,src}(\vec{r})$ (dipole) delta-correlated with respect to spatial position and polarization (orientation). Such electric current $\delta\vec{j}^{\,src}(\vec{r})$ can be thought as sum of three mutually perpendicular and statistically independent random currents (dipoles). On this way we come to a model of random electric dipole source inside a biological object oriented parallel to coupled tuned linear wire antennas. We suppose the random dipole to be placed in near wave zone of both antennas and evaluate the antennas' excitation via this dipole with the aid of method elaborated by Brilliouin [24], Pistolkors [25], and Bechmann [26] to study electromagnetic field in near zone of linear wire antenna (see also [27]). This model of random electric dipole source reveals the interference – extreme properties for fluctuations of excited in antennas currents depending on relative positions of antennas and the random dipole source. The revealed interference – extreme properties are analyzed from viewpoint to reconstruct the random dipole source position via special arrangement of two antennas on biological object boundary surface or via special scanning the single antenna along this boundary surface.

The organization of the paper is as follows. In Sec.2 the general equations' system for volume currents' densities inside coupled receiving antennas near biological object boundary surface is written in terms of a single antenna electric field tensor T –scattering operator. In Sec.3 the derived general system is transformed to reduced system of equations for surface currents on perfectly conducting antenna's surfaces written in terms of a single antenna surface T- scattering operator. In Sec. 4 a passing to system equations for currents along coupled linear wire antennas is described and these equations' system is written in terms of a single wire T-scattering operator. In Sec. 5 the equations' system for currents along two coupled turned linear wire antennas is resolved in asymptotic limit of the so-called "big logarithm". In Sec. 6 the obtained currents' distributions along coupled tuned linear wire antennas is applied to the problem of antennas' exciting by a biological object thermal radiation via the object temperature homogeneous component. In Sec. 7 a local volume variation of the object temperature is modeled by corresponding random electric dipole source and the interference- extreme properties for fluctuations of excited in antennas' currents depending of relative positions of antennas and the random dipole source are considered. Conclusions are made in Sec. 8.

Coupled receiving antennas at biological object boundary

We start with the Helmholtz vector wave equation for electric field $\vec{E}(\vec{r})$ of a monochromatic electromagnetic wave of frequency ω in a three -dimensional ($3D$) inhomogeneous isotropic dielectric and conducting structure writing the equation as

$$\left[\delta_{\alpha\beta}\nabla^2 - \nabla_\alpha\nabla_\beta + \frac{\omega^2}{c^2}\hat{\varepsilon}_0(\vec{r}) - V(\vec{r})\,\delta_{\alpha\beta}\right]E_\beta(\vec{r}) = \frac{4\pi\omega}{ic^2}\,j_\alpha^{src}(\vec{r}) \qquad (1)$$

Our structure consists of homogeneous biological object occupying (Fig. 1) the left half space $y > 0$ of the Cartesian coordinate system x, y, z and receiving antennas placed at the object boundary surface $y = 0$. Symbol $\hat{\varepsilon}_0(\vec{r})$ denotes the complex dielectric permittivity of the structure without antennas (bounded biological object) equal to dielectric permittivity $\varepsilon_0 = 1$ in free space $y < 0$ and $\varepsilon = \varepsilon' + i\,(4\pi\sigma/\omega)$ inside biological object $y > 0$, with ε' and σ being real part of the complex dielectric permittivity and specific conductivity of the object, respectively. An effective "scattering potential" $V(\vec{r})$ of antennas is defined by

$$V(\vec{r}) = -\frac{\omega^2}{c^2}\sum_q(\hat{\varepsilon}_A - \varepsilon_0)\chi(\vec{r} - \vec{r}_q) \equiv \sum_q V_q(\vec{r}) \qquad (2)$$

where $\hat{\varepsilon}_A = \varepsilon_A + i\,(4\pi\sigma_A/\omega)$ is the complex dielectric permittivity of antenna,

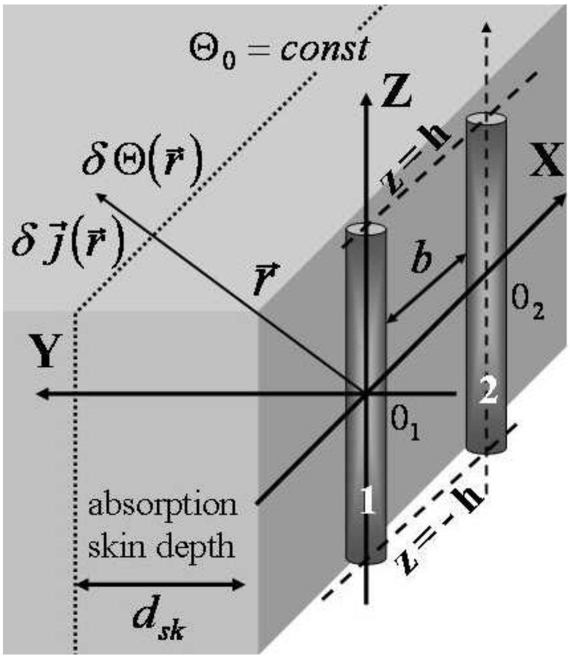

Figure 1. Schematic showing of biological object (grey area; dotted line symbolically visualizes an absorption skin depth d_{sk}) thermal radiation receiving on two coupled antennas whose length, diameter and the distance along the x-axis direction are denoted as $D = 2h$, $2a$, and b respectively; $a << D$. The designations Θ_0, $\delta\Theta(\vec{r})$, and $\delta\,\vec{j}^{\,src}(\vec{r})$ stand for homogeneous and random local volume variations of the object temperature $\Theta(\vec{r})$, and local electric current density variation related to the local temperature variation $\delta\,\Theta(\vec{r})$, respectively.

with ε_A and σ_A being its real dielectric permittivity and specific conductivity, respectively. The function $\chi(\vec{r} - \vec{r}_q)$ is the characteristic function of the q*-th* antenna, equal to unity if point $\vec{r}$ belongs to the region occupied by the q*-th* antenna and equal to zero otherwise. The antennas are assumed to be identical and centered at positions $\vec{r}_q$, $q = 1,2,3,\ldots$. Each antenna is characterized by a single scattering potential $V_q(\vec{r})$, defined via Eq.(2). The magnetic permittivity is supposed to be $\mu = 1$ everywhere. The vector $\vec{j}^{\,src}(\vec{r})$ denotes the electromagnetic field source volume current density, which can be random as in the case of biological object thermal radiation. The summation over repeated Greek subscripts is implied in the limits from 1 to 3, with 1, 2, 3 corresponding to the x, y, z axes. The Gaussian system of units is used and c denotes the light speed in the free space.

We are interested in the electric currents excited inside antennas. Let us introduce a vector

$$\vec{J}^{(q)}(\vec{r}) = V_q(\vec{r})\,\vec{E}(r) \equiv \frac{4\pi\omega}{ic^2}\left[\sigma_A + \frac{\omega}{4\pi i}(\hat{\varepsilon}_A - \varepsilon_0)\right]\chi(\vec{r} - \vec{r}_q)\vec{E}(\vec{r}) \equiv \frac{4\pi\omega}{ic^2}\,\vec{j}^{\,(q)}(\vec{r}) \qquad (3)$$

that presents accurate with the factor $4\pi\omega / ic^2$ a sum $\vec{j}^{\,(q)}(\vec{r})$ of volume conducting and displacement current densities inside the q-th antenna. A complete current density $\vec{J}(\vec{r}) = \sum_q \vec{J}^{(q)}(\vec{r})$ excited in all antennas can be evaluated, provided the electric field tensor T-scattering operator $T_{\alpha\beta}(\vec{r},\vec{r}\,')$ [16,17] of antennas' system is known . Really, denote $G^0{}_{\alpha\beta}(\vec{r},\vec{r}\,')$ the electric field retarded Green tensor function of the bounded biological object, which satisfies the Helmholtz Eq.(1) without antenna scattering potential in the left-hand side (LHS) and with the delta-source term $\delta_{\alpha\beta}\delta(\vec{r}-\vec{r}\,')$ in the right-hand side (RHS) of the equation and the radiation conditions in the infinity. The electric field Green tensor function G^0 enables one to bring [15-17] a solution problem for the Helmholtz Eq.(1) to wave integral equation

$$E_\alpha(\vec{r}) = E^0{}_\alpha(\vec{r}) + \int d\vec{r}\,' G^0_{\alpha\beta}(\vec{r},\vec{r}\,')\, V(\vec{r}\,')\, E_\beta(\vec{r}\,') \tag{4}$$

Here $\vec{E}^0(\vec{r})$ denotes the incident on antennas electric field given by

$$E^0{}_\alpha(\vec{r}) = \frac{4\pi\omega}{ic^2}\int d\vec{r}\,' G^0_{\alpha\beta}(\vec{r},\vec{r}\,')\, j^{src}_\beta(\vec{r}\,') \tag{5}$$

Solution to the wave integral Eq.(4) is presented [15-17] in terms of the electric field tensor T-scattering operator of antennas' system by an equality that has a form

$$E_\alpha(\vec{r}) = E^0_\alpha(\vec{r}) + \int d\vec{r}\,' \int d\vec{r}\,'' G^0_{\alpha\beta}(\vec{r},\vec{r}\,') T_{\beta\gamma}(\vec{r}\,',\vec{r}\,'')\, E^0_\gamma(\vec{r}\,'') \tag{6}$$

Comparison of this equality with integral Eq.(4) gives the Lippmann- Schwinger equation for the electric field T- scattering operator of antennas' system

$$T_{\alpha\beta}(\vec{r},\vec{r}\,') = V(\vec{r})\,\delta_{\alpha\beta}\delta(\vec{r}-\vec{r}\,') + V(\vec{r})\int d\vec{r}\,'' G^0_{\alpha\gamma}(\vec{r},\vec{r}\,'') T_{\gamma\beta}(\vec{r}\,'',\vec{r}\,') \tag{7}$$

and a symbolic operator relation $J \equiv VE = T\,E^0$, which in details is written as

$$J_\alpha(\vec{r}) = \int d\vec{r}\,'\, T_{\alpha\beta}(\vec{r},\vec{r}\,')\, E^0_\beta(\vec{r}\,') \tag{8}$$

The obtained relation shows that one can indeed evaluate the complete current density excited in all antennas, having known the electric field tensor T- scattering operator of antennas' system and the incident on antennas electric field.

The wave integral Eq. (4) can be derived with the aid of vector Green theorem and the boundary conditions on antennas surfaces and at infinity, with similar to [16] manipulating for the case of bodies' system in free space.

Side by side with the electric field T-scattering operator of antenna's system it is convenient to consider also the electric field T-scattering operator of a single q-*th* antenna T_q, that satisfies the Lippmann-Schwinger equation with the single scattering potential, $T_q = V_q + V_q G^0 T_q$. According to Watson composition rule [12,15] connection between T- scattering operator of antenna's system and single scattering operators T_q of antennas is given by a system of operator equations

$$T = \sum_q T^{(q)}; \quad T^{(q)} = T_q + T_q\, G^0 \sum_{q' \neq q} T^{(q')} \tag{9}$$

Here $T^{(q)}$ has sense of T- scattering operator of q-*th* antenna coupled with all other antennas. Convolving Eq.(9) with the incident electric field E^0 gives for current densities $J^{(q)} = T^{(q)} E^0$ inside antennas (3) a basic system of equations

$$J_\alpha^{(q)}(\vec{r}) = J_{q\alpha}(\vec{r}) + \int d\vec{r}' \int d\vec{r}'' T_{q\alpha\beta}(\vec{r}, \vec{r}')\, G^0_{\beta\gamma}(\vec{r}', \vec{r}'') \sum_{q' \neq q} J_\gamma^{(q')}(\vec{r}'') \tag{10}$$

written in terms of current densities inside of single antennas $J_q = T_q\, E^0$ and the single scattering operators T_q of antennas.

The basis system of Eqs .(10) for currents' densities inside antennas is too complicated, having accounting the coupling effect not only between antennas but also between antennas and bounded biological object. In the next section the system (10) is simplified via accounting the coupling effect between antennas only.

Coupled receiving perfectly conducting antennas inside the object boundary subsurface

Remind that in practice [2-5] the mean size of contact antennas conforms to wavelength λ_1 inside a biological object. We use that conformity to avoid the problem of coupling between antennas and biological object boundary, with replacing in all equations of the preceding section the electric field Green tensor function $G^0{}_{\alpha\beta}(\vec{r}, \vec{r}')$ of the bounded biological object to the Green tensor function

$G^0{}_{\alpha\beta}(\vec{r}-\vec{r}')$ of unbounded biological object. Simultaneously one needs replacing in Eqs. (2) and (3) the permittivity ε_0 to ε.

We consciously use such rather rough approach to focus our attention on coupling effects between antennas at receiving the biological thermal radiation.

The above electric field Green tensor function of unbounded biological object is given by

$$G^0{}_{\alpha\beta}(\vec{r}-\vec{r}') = \left(\delta_{\alpha\beta} + \nabla_\alpha \nabla_\beta / k_1^2\right) G_0(\vec{r}-\vec{r}') \equiv \frac{ic^2}{4\pi\omega} L_{\alpha\beta}(\vec{r})\, G_0(\vec{r}-\vec{r}') \quad (11)$$

where $G_0(r) = \exp(ik_1 r)/(-4\pi r)$ denotes a scalar Green function. The basic Eqs.(10) for currents densities $\vec{j}^{(q)}(\vec{r}_q)$ inside two coupled antennas $q = 1,2$ takes a form

$$\frac{4\pi\omega}{ic^2} j_\alpha^{(1)}(\vec{r}_1) = \int d\,\vec{r}_1^{\,\prime}\, T_{1\alpha\beta}(\vec{r}_1, \vec{r}_1^{\,\prime}) \left[E_\beta^0(\vec{r}_1^{\,\prime}) + \int d\,\vec{r}_2^{\,\prime\prime}\, L_{\beta\gamma}(\vec{r}_1^{\,\prime}) G_0(\vec{r}_1^{\,\prime} - \vec{r}_2^{\,\prime\prime})\, j_\gamma^{(2)}(\vec{r}_2^{\,\prime\prime})\right] \quad (12)$$

and

$$\frac{4\pi\omega}{ic^2} j_\alpha^{(2)}(\vec{r}_2) = \int d\,\vec{r}_2^{\,\prime}\, T_{2\alpha\beta}(\vec{r}_2, \vec{r}_2^{\,\prime}) \left[E_\beta^0(\vec{r}_2^{\,\prime}) + \int d\,\vec{r}_1^{\,\prime\prime}\, L_{\beta\gamma}(\vec{r}_2^{\,\prime}) G_0(\vec{r}_2^{\,\prime} - \vec{r}_1^{\,\prime\prime})\, j_\gamma^{(1)}(\vec{r}_1^{\,\prime\prime})\right] \quad (13)$$

where indices 1 and 2 are related to antennas 1 and 2 (Fig.1), respectively, and integrations are performed inside volumes of these antennas.

The next problem consists in evaluating the single scattering operators T_1 and T_2 of antennas 1 and 2. Instead of direct solution to the Lippmann- Schwinger equation for a single scattering operator mentioned before a system of Eqs.(9), we remind an expression after Eqs.(10) for current density inside a single scattering antenna in terms of its single scattering operator and incident electric field, with rewriting the total electric field (6) everywhere around and inside antenna as

$$E_\alpha(\vec{r}) = E^0{}_\alpha(\vec{r}) + \int d\vec{r}'\, G^0_{\alpha\beta}(\vec{r}-\vec{r}')\; J_{1\beta}(\vec{r}') \quad (14)$$

Resolving this equation with respect to current density $\vec{J}_1(\vec{r})$ gives a convenient tool to find the single scattering operator Green theorem of perfectly conducting antenna. But previously to make so we need discussing some general properties of Eq. (14) for a body with finite dielectric permittivity.

The integral Eq. (14) for electric field $\vec{E}(\vec{r})$, with replacing $\vec{J}_1(\vec{r})$ to $V_1(\vec{r})\vec{E}(\vec{r})$, is known in wave multiple scattering theory for a long period of time and often related to pioneering papers of Foldy [28] and Lax [29]. Nevertheless, an opinion exists [30] that one needs verifying consistence the Eq. (14) with boundary conditions on the body surface directly. In this connection we would like to make the two following remarks.

First, as was mentioned after Eq, (8), the Eq. (14) has been derived [16] with the aid of vector Green theorem and accounting the boundary conditions.

Second, Eq. (14) is a three- dimensional singular integral equation and demands some accuracy at handling with it. If point $\vec{r}$ is placed inside the body one has to take into account the strong singularity of the tensor Green function, Eq. (11), at $\vec{r}\,' \to \vec{r}$, with decomposing [31-33] the one into a delta Dirac function term $(1/k_1^2)\,a_{\alpha\beta}\,\delta(\vec{r}-\vec{r}\,')$ and principal part $PS\,G^0_{\alpha\beta}(\vec{r}-\vec{r}\,')$, where a constant tensor $a_{\alpha\beta}$ depends on the shape of the exclusion domain chosen to define the principal part. If the exclusion domain is an infinitesimal sphere the tensor $a_{\alpha\beta} = \delta_{\alpha\beta}/3$. Substituting the decomposed tensor Green function into Eq. (14) gives after [32,34] an equation

$$F_\alpha(\vec{r}) = E^0{}_\alpha(\vec{r}) + Lim_{a\to 0} \int\limits_{|\vec{r}-\vec{r}\,'|\geq a} d\vec{r}\,' G^0_{\alpha\beta}(\vec{r}-\vec{r}\,')\hat{V}(\vec{r}\,')F_\beta(\vec{r}\,') \qquad (15)$$

where integration in the RHS is performed over body volume, with excluding an infinitesimally small spherical domain around point $\vec{r}$. A local electric field $\vec{F}(\vec{r})$ and transformed scattering potential $\hat{V}(\vec{r})$ are defined by

$$\vec{F}(\vec{r}) = \frac{\hat{\varepsilon}_A + 2\varepsilon}{3\varepsilon}\vec{E}(\vec{r}); \qquad \hat{V}(\vec{r}) = -3k_1^2\,\frac{\hat{\varepsilon}_A - \varepsilon}{\varepsilon_A + 2\varepsilon}\chi(\vec{r}-\vec{r}_1) \qquad (16)$$

The strong singular term of the tensor Green function in Eq. (11) at $\vec{r}\,'=0$ and $r\to 0$ has a form $(\delta_{\alpha\beta}-3\hat{r}_\alpha\hat{r}_\beta)/r^3$ and becomes of zero value by averaging over unit vector $\hat{r}$ along vector $\vec{r}$ directions. Hence the integral in the sense of principal value in the RHS of Eq. (15) is good defined one and can be expressed according to general theory of many dimensional singular integrals [35] as sum of absolutely converged integrals. However, Eq. (15) does not show directly its consistence with boundary conditions. Therefore we transform this equation to a physically transparent form usual for theory of electromagnetism [27].

Let us apply a lemma (see, e.g., [36], appendix 50) as the rule to bring out the second derivative outside the three dimensional singular integral

$$Lim_{a\to 0}\int\limits_{|\vec{r}-\vec{r}\,'|\geq a} d\vec{r}\,'\nabla_\alpha\nabla_\beta G_0(\vec{r}-\vec{r}\,')\,a(\vec{r}\,') = \nabla_\alpha\nabla_\beta Lim_{a\to 0}\int\limits_{|\vec{r}-\vec{r}\,'|\geq a} d\vec{r}\,' G_0(\vec{r}-\vec{r}\,')\,a(\vec{r}\,')-\frac{1}{3}\delta_{\alpha\beta}\,a(\vec{r}) \qquad (17)$$

where $a(\vec{r})$ is some scalar test function. This lemma enables one to transform Eq.(15) as follows

$$\vec{E}(\vec{r})=\vec{E}_0(\vec{r})+(k_1^2+\nabla\text{div})\vec{\Pi}(\vec{r}) \qquad (18)$$

Here $\vec{\Pi}(\vec{r})$ denotes the Hertz vector that is written in terms of the body polarization vector $\vec{P}(\vec{r})=[(\hat{\varepsilon}_A-\varepsilon)/4\pi]\vec{E}(\vec{r})$ as

$$\vec{\Pi}(\vec{r})=\frac{1}{\varepsilon}\int d\,\vec{r}\,'\frac{\exp(ik_1|\vec{r}-\vec{r}\,'|)}{|\vec{r}-\vec{r}\,'|}\vec{P}(\vec{r}\,') \qquad (19)$$

with omitting the limit symbol at the RHS integral. The scalar electric potential $\varphi(\vec{r})=-\text{div}\,\vec{\Pi}(\vec{r})$ is evaluated by applying the divergence operator under integral sign in the RHS of Eq. (19). Then similar to the case of dielectric polarization in electrostatics (see [27], paragraph 3.13) one can introduce a volume density $\rho(\vec{r})=-(1/\varepsilon)\,\text{div}\,\vec{P}(\vec{r})$ of body electric polarization charge for point $\vec{r}$ inside the body and a surface $\sigma(\vec{r}_S)=(1/\varepsilon)\,\vec{n}\,\vec{P}(\vec{r}_S)$ density one for point $\vec{r}_S$ on the body surface where $\vec{n}$ is the outward unit normal vector to the surface. The surface charge is coursed by polarization vector discontinuity at the body surface crossing and leads to discontinuity of the electric field (see [27], paragraph 3.15) on the body surface according the equation

$$\vec{E}_+ - \vec{E}_- = 4\pi\sigma\,\vec{n} \tag{20}$$

where the subscript plus and minus are related to regions out and inside the body at surface point $\vec{r}_S$, respectively. Eq. (20) means discontinuity the normal component of electric field at the body surface crossing according to relation $\varepsilon\,\vec{n}\,\vec{E}_+ = \hat{\varepsilon}_A\vec{n}\,\vec{E}_-$ as well as continuity of electric field tangential component at the body surface crossing.

Having verified the consistence of Eq. (14) with the boundary conditions on the body surface, let us return to system Eqs. (12) and (13) for currents' densities inside two coupled antennas. In the limit of a perfectly conducting antenna the current density inside its volume becomes to be confined near the antenna surface S. Therefore we write, e.g., $\vec{j}^{(1)}(\vec{r})d\,\vec{r} = \vec{i}^{(1)}(\vec{r})\;dS$ where $d\,\vec{r}$ and $d\,S$ are elements of antenna volume and surface, respectively, as well as $\vec{j}^{(1)}(\vec{r})$ and $\vec{i}^{(1)}(\vec{r})$ are antenna volume and surface current densities, respectively. We write similarly a relation $T_{\alpha\beta}(\vec{r},\vec{r}\,')\,d\,\vec{r}\,d\vec{r}\,'$ = $t_{\alpha\beta}(\vec{r},\vec{r}\,')\;dS\,dS'$ defined a surface T - scattering operator $t_{\alpha\beta}(\vec{r},\vec{r}\,')$. Substituting into Eqs. (12) and (13) gives

$$\frac{4\pi\omega}{ic^2}i_\alpha^{(1)}(\vec{r}_1) = \int d\,S_1'\;t_{1\alpha\beta}(\vec{r}_1,\vec{r}_1\,') \left[E_\beta^0(\vec{r}_1\,') + \int d\,S_2''\;L_{\beta\gamma}(\vec{r}_1\,')G_0(\vec{r}_1\,' - \vec{r}_2\,'')i_\gamma^{(2)}(\vec{r}_2\,'')\right] \tag{21}$$

and

$$\frac{4\pi\omega}{ic^2}i_\alpha^{(2)}(\vec{r}_2) = \int d\,S_2'\;t_{2\alpha\beta}(\vec{r}_2,\vec{r}_2\,') \left[E_\beta^0(\vec{r}_2\,') + \int d\,rS_1''\;L_{\beta\gamma}(\vec{r}_2\,')G_0(\vec{r}_2\,' - \vec{r}_1\,'')i_\gamma^{(1)}(\vec{r}_1\,'')\right] \tag{22}$$

Here the indices 1 and 2 are related to antennas 1 and 2, respectively, but in deference from Eqs.(12) and (13) the integrations are performed now along the surfaces of antennas 1 and 2. Because the total electric field $\vec{E}(\vec{r})$ on perfect conducting antenna's surface is equal to zero the Eq.(18) takes a form

$$E^0{}_\alpha(\vec{r}) + L_{\alpha\beta}(\vec{r})\int d\,S'\,G_0(\vec{r}-\vec{r}')\;i_{1\beta}(\vec{r}') = 0 \qquad (23)$$

if point $\vec{r}$ is placed on the antenna 1 surface. Resolving Eq.(23) with respect to the surface current density and bearing in mind a relation

$$\frac{4\pi\omega}{ic^2}\;i_\alpha^{(1)}(\vec{r}_1) = \int d\,S_1'\;\;t_{1\alpha\beta}(\vec{r}_1,\vec{r}_1')\;\;E_\beta^0(\vec{r}_1') \qquad (24)$$

one can get the desired single surface T - scattering operator $t_{1\alpha\beta}(\vec{r},\vec{r}')$ of antenna 1.

Coupled receiving linear wire antennas

Consider the case of linear wire antennas in the form of thin vibrator- dipoles to be parallel to z axis and occupying the regions $-h \le z \le h$ as depicted in Fig.1. For a cylindrical vibrator with diameter $2a$ only tangential component $i_z(z)$ of the surface current density $\vec{i}\;(z)$ exists and one can introduce a total current $I(z) = 2\pi\,a\,i_z(z)$ in the cross-section z of the vibrator. Eq.(23) for the surface current density of a single antenna is transformed in the case of linear wire antenna under consideration to the Pocklington integral equation [20] (see also [37])

$$\left(\frac{d^2}{dz^2} + k_1'^2\right)A_z(r=a,z) = i\omega\varepsilon'\,E^0{}_z(r=a,z) \qquad (25)$$

with vector potential z -component $A_z\,(r,z)$ given by

$$A_z(r,z) = \frac{1}{2\pi}\int_0^{2\pi} d\phi' \int_{-h}^{h} dz'\;\;\frac{\exp(ik_1'\,R)}{R}\;\;I(z') \qquad (26)$$

Here $R = |\,\vec{r}-\vec{r}'\,|$ as well as the observation $\vec{r} = (r,\phi,z)$ and source $\vec{r}' = (r',\phi',z')$ points are presented in cylindrical coordinates. The z - component of vector potential (26) is evaluated asymptotically for thin vibrator near zone, where $k_1' r << 1$ and $R \approx |\,z - z'\,|$, as a sum of logarithm's and addition terms

$$A_z(r,z) \approx -2\,I(z)\ln(k_1'\,r) + W[I,z] \qquad (27)$$

with addition term $W[I,z]$ having a current functional form

$$W[I,z]=\int_{-h}^{h} d\,z'\left[\frac{z-z'}{|z-z'|}\frac{d\,I(z')}{d\,z'}-ik_1' I(z'')\right]\exp(i\,k_1'|z-z'|)\ln(2k_1'|z-z'|) \quad (28)$$

Substituting the vector potential z - component asymptotics (27) into Pocklington integral Eq.(25) leads to the Leontovich – Levin version [18] of Hallen integro-differential equation [19] (see also [37]) for current distribution $I(z)$ along single linear wire thin vibrator-dipole receiving antenna

$$\frac{d^2 I}{dz^2}+k_1'^2\, I=-i\omega\varepsilon'\eta\left\{E_z^0(z)+G[I,z]\right\};\quad I(\pm h)=0 \quad (29)$$

where current functional $G[I,z]$ is defined by

$$-\,i\omega\varepsilon'\,G[I,z]=\left(\frac{d^2}{dz^2}+k_1'^2\right)W[I,z] \quad (30)$$

Remind that $\eta=1/(2\ln k_1'a)$ is small parameter in asymptotic limit of “big logarithm”.

Relation (24) between a single antenna surface current density and antenna single surface T - scattering operator takes in the limit of linear wire thin vibrator – dipole antenna the form

$$\frac{4\pi\omega}{ic^2}I(z)=\int_{-h}^{h} dz'\; t\,(z,z')\,E_z^0(z') \quad (31)$$

with a single wire T - scattering operator $t\,(z,z')$ defined by

$$t\,(z,z')=\int_0^{2\pi} a\,d\phi\int_0^{2\pi} a\,d\phi'\,t_{1zz}(z,\phi;z',\phi')) \quad (32)$$

Similarly, the system of Eqs. (21) and (22) for surface current densities of two coupled antennas passes to equation system for current $I^{(q)}(z_q)$ distributions along two coupled linear wire antennas q = 1,2 written in terms of single wire T - scattering operators $t_q\,(z_q,z_q')$ as

$$\frac{4\pi\omega}{ic^2} I^{(1)}(z_1) = \int_{-h}^{h} dz_1' t_1(z_1, z_1') \left[E_z^0(z_1') + L_{zz}(z_1') \int_{-h}^{h} dz_2'' G_0(z_1', z_2'') I^{(2)}(z_2'') \right] \quad (33)$$

and

$$\frac{4\pi\omega}{ic^2} I^{(2)}(z_2) = \int_{-h}^{h} dz_2' t_1(z_2, z_2') \left[E_z^0(z_2') + L_{zz}(z_2') \int_{-h}^{h} dz_1'' G_0(z_2', z_1'') I^{(1)}(z_1'') \right] \quad (34)$$

Here $G_0(z_1, z_2) = \exp(ik_1' R_{12})/(-4\pi R_{12})$ where $R_{12}^2 = b^2 + (z_1 - z_2)^2$ and b is the distant between antennas (Fig. 1). In the next section we present asymptotic solution to Leontovich – Levin-_Hallen Eq.(29) and to equation system (33) and (34).

Current distributions along two coupled tuned receiving linear wire antennas in the limit of "big logarithm"

Bearing in mind the small parameter η one can apply after [18] the perturbation method of solution to Eq.(29) for current distribution $I(z)$ along single linear wire antenna in the form of expansion $I = I_0 + \eta\, I_1 + \eta^2\, I_2 + \cdots$ with result of substitution

$$\left(\frac{d}{dz^2} + k_1'^2 \right) I_0(z) = 0; \quad I_0(\pm h) = 0 \quad (35)$$

$$\left(\frac{d}{dz^2} + k_1'^2 \right) I_1(z) = -i\omega\varepsilon' \left\{ E_z^0(z) + G[I, z] \right\}; \quad I_1(\pm h) = 0 \quad (36)$$

This system of equations should be resolved successively. We restrict ourselves by simple case of tuned vibrator-dipole.

The length $2h$ of tuned vibrator – dipole is equal to the whole multiple of half wavelength $\lambda_1/2$ in the biological object, $k_1' h = n\pi/2$ or $2h = n\lambda_1/2$, with n being integer. Eq.(35) for zero approximation has a solution $I_0(z) = I_0\, \psi_n(z)$, where I_0 is current amplitude and function $\psi_n(z) = \cos(k_1' z)$ if n is odd and $\psi_n(z) = \sin(k_1' z)$ if n is even. Current amplitude I_0 is defined by orthogonality condition of zero approximation to RHS of Eq.(36) for the first approximation that after some algebra gives

$$I_0 = \frac{1}{Z_1} \int_{-h}^{h} dz\, \psi_n(z)\, E_z^0(z) \qquad (37)$$

The quantity Z_1 has sense of input impedance of single vibrator – dipole feeding in the current maximum point, when $E_z^0(z) = v_0 \delta(z - z_0)$ and $\psi_n(z_0) = 1$, and it is evaluated via formula

$$\frac{\omega \varepsilon'}{k_1'} Z_1 = Di\left(2\pi n\right) - i\, Si\left(2\pi n\right) \qquad (38)$$

Here the functions $Si(x)$ and $Di(x)$ are defined by integrals

$$Si(x) = \int_0^x dt \frac{\sin t}{t}; \quad Di(x) = \int_0^x dt \frac{1 - \cos t}{t} \qquad (39)$$

with $Si(x)$ being the integral sine [38] and regular function $Di(x)$ related to the integral cosine $Ci(x)$ and Euler constant $C \approx 0.5772$ as $Di(x) = \ln x + C - Ci(x)$.

Substituting current distribution $I_0(z)$ along turned single vibrator- dipole into Eq.(31) gives for single wire T - scattering operator $t(z, z')$ of such vibrator-dipole a separable value defined by

$$\frac{ic^2}{4\pi\omega} t(z, z') = \frac{1}{Z_1} \psi_n(z) \psi_n(z') \qquad (40)$$

The separability property of single wire T - scattering operator makes the Eqs.(33) and (34) for current distributions along two coupled linear wire antennas to be exactly resolved. One can write out result of this resolution in a form similar to the case of a single linear wire antenna putting $I^{(q)}(z_q) = I_0^{(q)} \psi_n(z_q)$ where amplitudes $I_0^{(q)}$ of current distribution along two coupled linear wire antennas are given by

$$I_0^{(1)} = \frac{1}{1 - a_{12}^2} \frac{1}{Z_1} \left[\int_{-h}^{h} dz_1 \psi_n(z_1)\, E_z^0(z_1) + a_{12} \int_{-h}^{h} dz_2 \psi_n(z_2)\, E_z^0(z_2) \right] \qquad (41)$$

and

$$I_0^{(2)} = \frac{1}{1-a_{12}^2}\frac{1}{Z_1}\left[\int_{-h}^{h} dz_2 \psi_n(z_2)\, E_z^0(z_2) + a_{12}\int_{-h}^{h} dz_1 \psi_n(z_1)\, E_z^0(z_1)\right] \qquad (42)$$

Here the indices 1 and 2 are related to antennas 1 and 2, respectively, and integrations are performed along these two antennas. A specific coupling factor $a_{12} = -Z_{12}/Z_1$ where Z_{12} is mutual impedance [37] of two vibrator - dipoles is defined by integral equality

$$\frac{i\omega\varepsilon'}{4\pi} Z_1 a_{12} = \int_{-h}^{h} dz_1 \psi_n(z_1)\left(k_1'^2 + \frac{\partial^2}{\partial z_1^2}\right)\int_{-h}^{h} dz_2 G_0(z_1, z_2)\,\psi_n(z_2) \qquad (43)$$

Transforming double integral in this equality RHS by the method [25-26], with applying part by part integration, leads to a working formula for coupling factor evaluation

$$\frac{\omega\varepsilon'}{k_1'} Z_1 a_{12} = f(x_+) + f(x_-) - 2\, f(x_0) \qquad (44)$$

where a function $f(x\ )$ is expressed in terms of integral cosine and integral sine as $f(x\ ) = Ci(x) + i\, Si(x)$ and parameters $x_\pm$ and x_0 have a form

$$x_\pm = n\pi\left(\pm 1 + \sqrt{1 + \frac{b^2}{4h^2}}\right); \quad x_0 = n\pi\frac{b}{2h} \qquad (45)$$

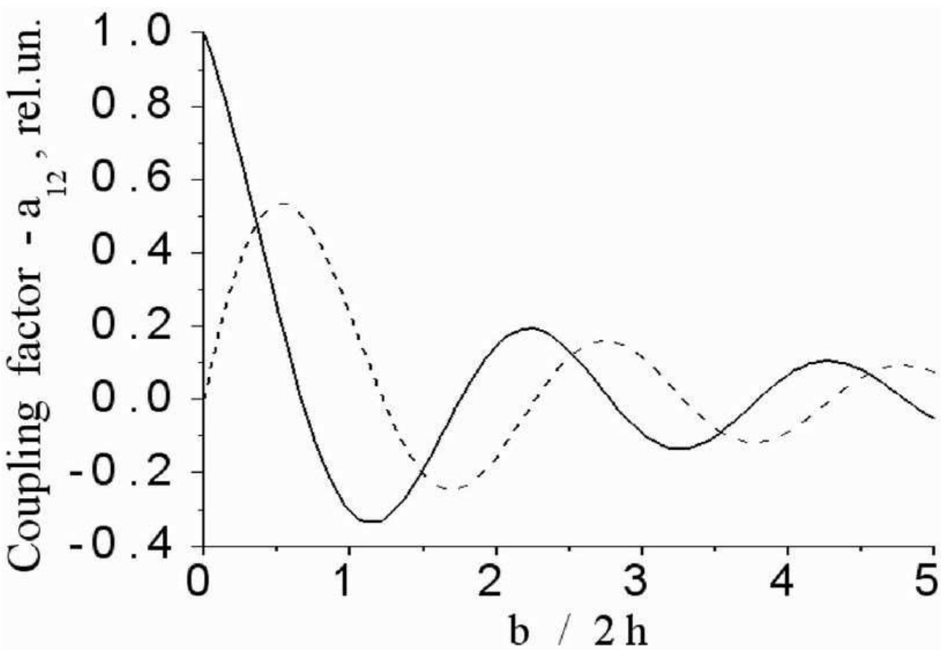

Figure 2. Dependence of the real (solid line) and imaginary (dashed line) parts of the specific coupling factor $-a_{12} = Z_{12} / Z_1$ of two half wavelength $2h = \lambda_1 / 2$ antennas versus normalized distance $b / 2h$ between antennas.

An useful asymptotics for coupling factor a_{12} at small distances b between antennas reads

$$a_{12}\big|_{b/2h \to 0} \approx -1 - i\pi \frac{b}{h} \frac{k_1' / \omega\varepsilon'}{Z_1} \qquad (46)$$

where input impedance Z_1 of single vibrator – dipole is defined by Eq. (38). Fig.2 represents the real and imaginary parts of the coupling factor a_{12} versus the dimensionless distant $b / 2h$ between two half wavelength $2h = \lambda_1 / 2$ vibrators.

In the next section we apply the presented currents' distributions along single tuned vibrator- dipole as well as along two coupled tuned vibrator – dipoles to the problem of antennas' exciting by a biological object thermal radiation via the object temperature homogeneous component.

Exciting single and coupled tuned receiving linear wire antennas by equilibrium thermal radiation

According to the general theory [6] the electromagnetic thermal radiation of a heated absorbing body is created by a random electric volume current density $\vec{j}^{\,src}(\vec{r})$ with the spatial correlation function spectral density given by

$$\left\langle j_\alpha^{\,src}(\vec{r})\, j_\beta^{\,src*}(\vec{r}\,') \right\rangle = \frac{1}{4\pi^2} \omega\varepsilon''(\vec{r})\Theta(\vec{r})\,\delta_{\alpha\beta}\delta(\vec{r} - \vec{r}\,') \qquad (47)$$

where $\Theta(\vec{r})$ denotes the body temperature multiplied by the Boltzmann constant. In this section we are interesting in effects of homogeneous component Θ_0 of the biological object temperature. In this case one obtains from Eqs. (5), (11) and (47) the following expression for the incident on antennas random electric field spatial correlation function spectral density

$$\left\langle E_\alpha^0(\vec r)E_\beta^{0*}(\vec r')\right\rangle = -4\frac{\omega}{c^2}\Theta_0 \ \operatorname{Im} G_{\alpha\beta}^0(\vec r-\vec r') \quad (48)$$

that is a standard form [10] of equilibrium thermal radiation electric field spatial correlation function spectral density.

Apply first the expression in Eq. (48) to the case of a single tuned vibrator-dipole antenna exciting by a biological object thermal radiation via object temperature homogeneous component. Eq. (37) gives for the fluctuations' spectral density $\left\langle |I_0|^2\right\rangle$ of current distribution along receiving antenna amplitude an equality

$$\left\langle |I_0|^2\right\rangle = \frac{1}{|Z_1|^2}\int_{-h}^{h} dz\int_{-h}^{h} dz'\psi_n(z)\psi_n(z')\left\langle E_z^0(z)E_z^{0*}(z'')\right\rangle \quad (49)$$

where one sees in the RHS integrand the auto- correlation function of random electric field inside antenna. Using for this auto- correlation function the expression in Eq. (48) and transforming the double integral by abovementioned method [25-26] leads to

$$\left\langle |I_0|^2\right\rangle = \frac{\operatorname{Re} Z_1}{|Z_1|^2}\frac{\Theta_0}{\pi} \quad (50)$$

that is actually the Nyquist formula for thermal excitation in conductors [39].

Turn now to the problem of two coupled tuned vibrator-dipoles' exciting by a biological object thermal radiation via object temperature homogeneous component. On base of Eqs. (41), (42) and (48) the fluctuations' spectral density $\left\langle \left|I_0^{(1)}\right|^2\right\rangle$ of current distribution amplitude along, e.g., the antenna 1 is given by relation

$$\begin{aligned} &|Z_1|^2 \left|1-a_{12}^2\right|^2 \left\langle \left|I_0^{(1)}\right|^2\right\rangle \\ &= \left(1+|a_{12}|^2\right)\int_{-h}^{h} dz_1\int_{-h}^{h} dz_1'\psi_n(z_1)\psi_n(z_1')\left\langle E_z^0(z_1)E_z^{0*}(z_1')\right\rangle \\ &\quad + 2\left(\operatorname{Re} a_{12}\right)\int_{-h}^{h} dz_1\int_{-h}^{h} dz_2\,\psi_n(z_1)\psi_n(z_2')\left\langle E_z^0(z_1)E_z^{0*}(z_2)\right\rangle \end{aligned} \quad (51)$$

The first double integral in the RHS of this relation takes into account electric field auto-correlations along single antenna 1 and single antenna 2, as in the case of the Nyquist formula in Eq. (49) derivation, and was considered actually in [9]. In deference, the second double integral takes into account cross-correlation of electric field fluctuations along antenna 1 and antenna 2 and was missed in [9]. Proceeding evaluation of double integrals in the Eq. (51) RHS leads to the following result

$$\left\langle \left| I_0^{(1)} \right|^2 \right\rangle = \frac{\operatorname{Re} Z_1}{\left| Z_1 \right|^2} \frac{\Theta_0}{\pi} f_{12} \tag{52}$$

where factor f_{12} of equilibrium thermal radiation coherence between two coupled antennas has a real value and is defined by

$$f_{12} = \frac{1}{\left| 1 - a_{12}^2 \right|^2} \left[1 + \left| a_{12} \right|^2 - 2(\operatorname{Re} a_{12}) \frac{\operatorname{Re}(Z_1 a_{12})}{\operatorname{Re} Z_1} \right] \tag{53}$$

A part of this factor, which accounting for the term $1 + \left| a_{12} \right|^2$ only in the Eq. (53) RHS square brackets, was obtained in [9] on a phenomenological level of consideration, that is in terms of coupled antenna 1 input impedance Z_{1inp} = $Z_1 - (Z_{12}^2 / Z_1)$ and mutual impedance Z_{12}. Asymptotics in Eq. (46) for coupling factor a_{12} at small distances b between antennas shows that factor in Eq. (53) has a limit $f_{12} \to 1/4$ as distant between antennas becomes much smaller their length.

Exciting single and coupled tuned receiving linear wire antennas by thermal radiation from modeled local temperature inhomogeneity

In the preceding section a problem was considered about single and coupled tuned vibrator-dipoles' exciting by a biological object thermal radiation via object temperature $\Theta(\vec{r})$ homogeneous component Θ_0. One can relate to this temperature homogeneous component a random electric volume current density component $\vec{j}_0^{\,src}(\vec{r})$ in Eq. (47) LHS. Similarly we can connect the object temperature local volume spatial variation $\delta\Theta(\vec{r})$ with a corresponding local random electric volume current density variation $\delta \vec{j}^{\,src}(\vec{r})$. Supposition about statistical independence from $\vec{j}_0^{\,src}(\vec{r})$ leads to the spatial correlation function spectral density for local variation $\delta \vec{j}^{\,src}(\vec{r})$ in a form of Eq. (47), with replacing $\Theta(\vec{r})$ to $\delta\Theta(\vec{r})$ in the RHS of this equation that is

$$\left\langle \delta j_\alpha^{\;src}(\vec{r})\,\delta j_\beta^{\;src*}(\vec{r}\,')\right\rangle = \frac{1}{4\pi^2}\,\omega\varepsilon''(\vec{r})\,\delta\Theta(\vec{r})\,\delta_{\alpha\beta}\delta(\vec{r}-\vec{r}\,') \qquad (54)$$

Eqs. (5), (11) and (54) give expression for the incident on antennas random electric field $\delta \vec{E}^{\,0}(\vec{r})$ spatial correlation function spectral density caused by local spatial variation of the object temperature

$$\left\langle \delta E_\alpha^0(\vec{r})\,\delta E_\beta^{0*}(\vec{r}\,')\right\rangle = 4\frac{\omega^3}{c^2}\int d\,\vec{r}_1\,G_{\alpha\alpha'}^0(\vec{r}-\vec{r}_1)\,G_{\beta\alpha'}^{0*}(\vec{r}\,'-\vec{r}_1)\,\varepsilon''(\vec{r}_1)\,\delta\Theta(\vec{r}_1) \quad (55)$$

Detailing the idea about local random electric volume current density variation one can split this variation into sum $\delta \vec{j}^{\,src}(\vec{r}) = \sum_\alpha \delta \vec{j}_\alpha^{\,src}(\vec{r})$ of three mutually perpendicular and statistically independent random currents (random dipoles) $\delta \vec{j}_\varepsilon^{\,src}(\vec{r}) = \delta j_\alpha^{src}(\vec{r})\,\hat{e}_\alpha$ where $\hat{e}_\alpha$ are the unit vectors along the x, y, z axes, respectively to $\alpha = 1,2,3$. The random currents $\delta j_\alpha^{src}(\vec{r})$ satisfy the Eq. (54) evidently. Correspondingly the incident random electric field splits into sum $\delta \vec{E}^{\,0}(\vec{r}) = \sum_\gamma \delta \vec{E}_\gamma^{\,0}(r)$ of three statistically independent incident random electric fields defined by

$$\left(\delta \vec{E}_\gamma^{\,0}(\vec{r})\right)_\alpha = \frac{4\pi\omega}{ic^2}\int d\vec{r}\,'\,G_{\alpha\beta}^0(\vec{r}-\vec{r}\,')\;\delta j_\gamma^{src}(\vec{r}\,')(\hat{e}_\gamma)_\beta \qquad (56)$$

Physically the random electric field $\delta \vec{E}_\gamma^{\,0}(\vec{r})$ is created by random electric current (dipole source) $\delta \vec{j}_\gamma^{\,src}(\vec{r})$.

We have come up closely to our basic model of random electric dipole source inside a biological object oriented parallel to single (Fig. 3) or coupled (Fig. 1) tuned linear wire antennas. In this model only the random electric field $\delta \vec{E}_z^0(\vec{r})$ created by random electric current $\delta \vec{j}_z^{src}(\vec{r}')$ is taken into account. We think the last random electric current as being confined inside a thin and finite cylindrical region, which is extended in limits $z - \Delta z /2 \le z' \le z + \Delta z /2$ along the Z'-axes and defined by a

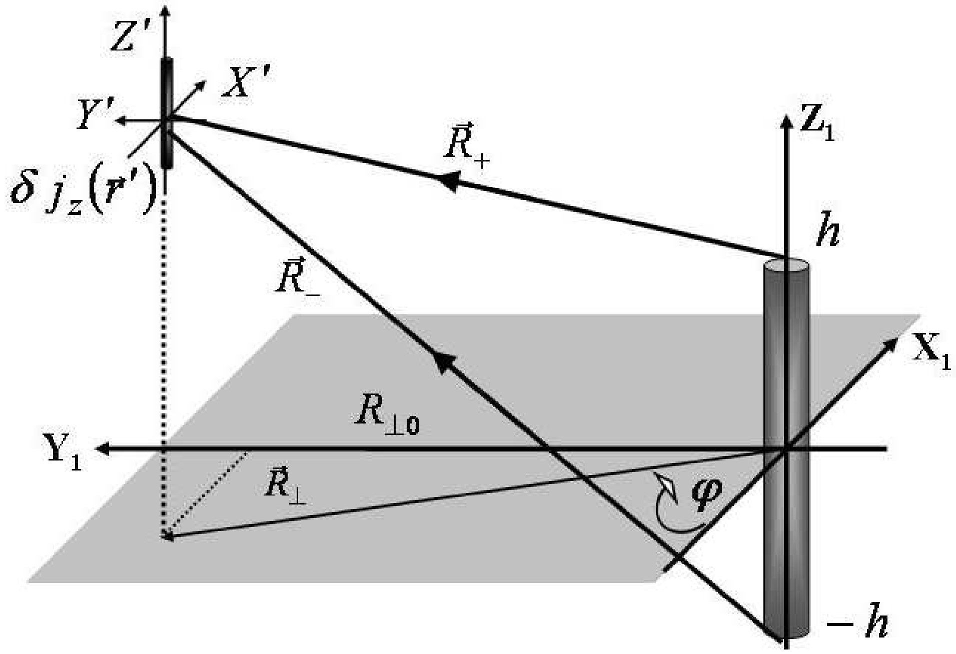

Figure 3. Schematic showing of vibrator antenna in thermal radiation field of random electric dipole source $\delta \vec{j}_z^{src}(\vec{r}')$ which is extended in limits $z - \Delta z /2 \le z' \le z + \Delta z /2$ along the Z'-axes and defined by a vector $\vec{R}_\perp$ in the X_1Y_1- plane (gray area) with length $R_\perp$ and azimuth angle φ.

vector $\vec{R}_\perp$ in the X_1Y_1- plane as, e.g., in the case of the single vibrator- dipole antenna (Fig.3). Integrating the random electric volume current density variation $\delta \vec{j}_z^{src}(\vec{r}')$ over the cylindrical region cross- section leads to a total random current variation $\delta I_z^{src}(z')$ in chosen cross-section, with a longitudinal correlation function spectral density giving by

$$\left\langle \delta I_z^{src}(z)\,\delta I_z^{src}(z') \right\rangle = \frac{1}{4\pi^2}\,\omega\varepsilon''\,\frac{\Delta\Omega}{\Delta z}\,\delta\,\Theta(z)\,\delta(z-z') \qquad (57)$$

Here $\Delta\Omega$ is a volume of the random electric dipole source.

One started actually to consider a problem of exciting single receiving linear wire antenna by thermal radiation from local temperature inhomogeneity. In the framework of the introduced random electric dipole source model, the z - component $\delta E_z{}^0(\vec{r})$ of the incident random electric field along single vibrator- dipole antenna (Fig. 3) is given according to Eqs. (11) and (56) by relation

$$\frac{i\omega\varepsilon'}{4\pi}\delta E_z^0(z_1) = \int d\,z'\left(k_1'^2 + \frac{\partial^2}{\partial z_1{}^2}\right)G_0(z_1,z')\,\delta I_z(z') \qquad (58)$$

where a Green function $G_0(z_1,z')$ is obtained from $G_0(z_1,z_2)$ defined after Eq.(34) by replacing z_2 to $z'$and b to $R_\perp$. The incident random electric field in Eq.(58) excites along single tuned vibrator- dipole receiving antenna a current distribution with random amplitude δI_0 given by Eq.(37), with replacing $E_z^0(z)$ to $\delta E_z^0(z)$ in the RHS integrand. Thus one gets

$$Z_1\delta I_0 = \frac{4\pi}{i\omega\varepsilon'}\int_{-h}^{h} dz_1\psi_n(z_1)\int d\,z'\left(k_1'^2 + \frac{\partial^2}{\partial z_1{}^2}\right)G_0(z_1,z')\,\delta I_z(z') \qquad (59)$$

Applying to the RHS of this equation the integration part by part with respect to variable z_1 , similarly with integral in Eq. (43), gives

$$Z_1\delta I_0 = -\frac{4\pi}{i\omega\varepsilon'}\int d\,z'\left[\psi_n'(h)G_0(h,z') - \psi_n'(-h)G_0(-h,z')\right]\delta I_z(z') \qquad (60)$$

For the case of tuned vibrator –dipole of length $2h = n\lambda_1/2$ equal to odd whole number n of half wavelengths Eq. (60) is transformed as

$$Z_1\delta I_0 = -(-1)^{(n-1)/2}\frac{k_1'}{i\omega\varepsilon'}\int d\,z'\left[\frac{\exp\left(i\,k_1R_+(z')\right)}{R_+(z')} + \frac{\exp\left(i\,k_1R_-(z')\right)}{R_-(z')}\right]\delta I_z(z') \qquad (61)$$

where $R_\pm(z')$ are defined by $R_\pm^2(z') = R_\perp^2 + (h \mp z')^2$ (see Fig.4). According to obtained equation the two spherical waves are propagated from a point z' of random electric dipole source towards receiving vibrator-dipole ends. Bearing in mind the reciprocity between receiving and transmitting antennas one can say also that two spherical waves are propagated from vibrator- dipole ends towards the random electric dipole source and interfere on the source area. Eqs. (57) and (61) enable us to write out for the fluctuations' spectral density $\left\langle \left|\delta I_0\right|^2 \right\rangle$ of current distribution along receiving antenna amplitude caused by random electric dipole source thermal radiation a following equality

$$\left(\frac{\omega\varepsilon'}{k_1'}\right)^2 \left|Z_1\right|^2 \left\langle \left|\delta I_0\right|^2 \right\rangle = \frac{1}{4\pi^2}\omega\varepsilon''\Delta\Omega\ \delta\Theta\,\overline{F}(z) \qquad (62)$$

A function $\overline{F}(z)$ in the RHS of this equality has a form of integral

$$\overline{F}(z) = \frac{1}{\Delta z}\int_{z-\Delta z/2}^{z+\Delta z/2} dz' F(z') \qquad (63)$$

where integrand $F(z)$ is defined by

$$F(z) = A_+^2(z) + A_-^2(z) + 2A_+(z)A_-(z)\cos\left[k_1'\left(R_+(z) - R_-(z)\right)\right] \qquad (64)$$

with $A_\pm(z) = \exp\left[-k_1''R_\pm(z)\right] / R_\pm(z)$. Similarly to optics [36] the definitions in Eqs.(63) and (64) can be called the single linear wire receiving antenna interference functions for the cases of finite extended and point random electric dipole source, respectively.

The spatial integral averaging in the RHS of Eq. (63) along random electric dipole source extension we perform using an approximate formula

$$\frac{1}{\Delta z}\int_{z-\Delta z/2}^{z+\Delta z/2} dz' \cos\left[a(z')\right] \approx \Gamma\left(\frac{da(z)}{dz}\frac{\Delta z}{2}\right)\cos\left[a(z)\right] \qquad (65)$$

where a function $\Gamma(z) = (\sin z)/z$. This formula is derived by writing $dz' = da(z')/(da(z')/dz')$ and approximate bringing the denominator $da(z')/dz'$ outside the integral in the middle point z. Actually we needs spatial averaging the fast varying cosine term of $F(z)$ only that gives for $\overline{F}(z)$ an expression

$$\overline{F}(z) \approx A_+^2(z) + A_-^2(z) + 2A_+(z)A_-(z)\Gamma\left[\frac{2k_1'h\,\Delta z}{R_+(z)+R_-(z)}\right]\cos\left[k_1'\left(R_+(z)-R_-(z)\right)\right] \tag{66}$$

As one sees the interference function $\overline{F}(z)$ for finite extended random electric dipole source is different from such interference function $F(z)$ for point source on a

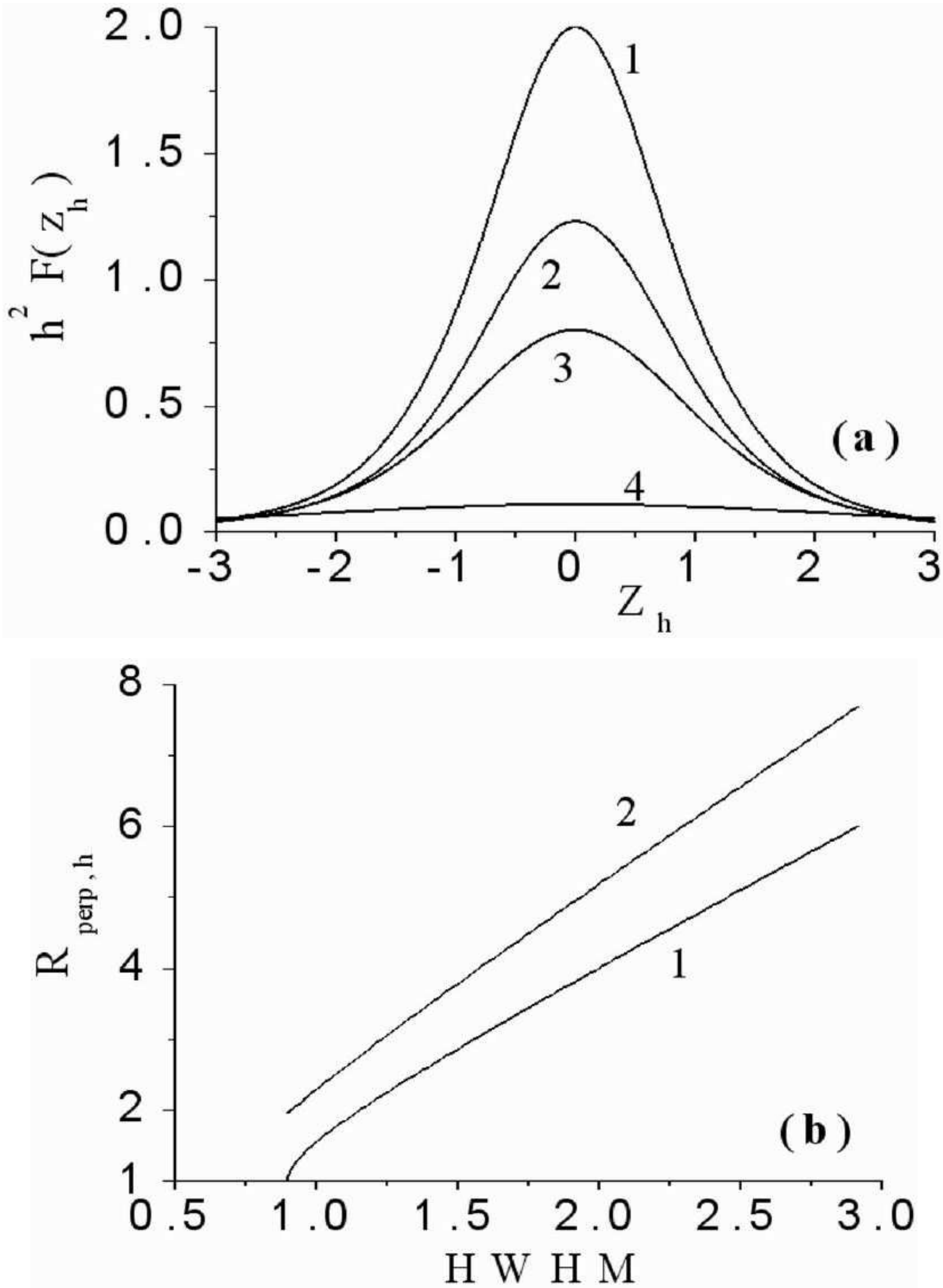

Figure 4. (**a**) The normalized interference function versus normalized height $z_h = z/h$ (from the X_1Y_1 plane) of random electric dipole point source at

different values of the point source normalized seeming depths $R_{\perp,h} = R_{\perp} / h$, with neglecting object absorption: $R_{\perp,h}$ = 1 (curve 1), 1.5 (2), 2 (3), and 6 (4). (**b**) the normalized seeming depth $R_{\perp,h}$ versus HWHM of the curves like ones in the panel (a) is delivered by curve 1 and calculated by Eq.(67) is given by curve 2.

Γ-factor that defines a spatial coherence degree of extended random electric dipole source.

Our next task consists in study the interference function $\overline{F}(z)$ extreme properties depending on random electric dipole source extension and $3D$ position defined by mentioned above coordinate z and vector $\vec{R}_{\perp}$. In this case the coordinate z gives height of random electric dipole source centre above ($z \geq 0$) or under ($z \leq 0$) the single linear wire antenna X_1, Y_1- plane (Fig. 3) while the vector $\vec{R}_{\perp}$ characterizes $2D$ position of random electric dipole source centre projection on the X_1, Y_1-plane that can be characterized also by the vector $\vec{R}_{\perp}$ length $R_{\perp}$ and its azimuth angle φ (see Fig. 3). The minimum value $R_{\perp 0}$ of the vector $\vec{R}_{\perp}$ length corresponds to the azimuth angle $\varphi = \pi / 2$ and gives us the real depth of random electric dipole source centre, with $R_{\perp}$ being a seeming depth of this source centre. Henceforth the aim of the single receiving antenna scanning along biological object boundary surface $y_1 = 0$ consists to get the random electric dipole source centre inside the X_1, Y_1-plane, first, and to define the source centre real depth by, e.g., placing this centre on the Y_1- axes, second. We intend to show that interference function $\overline{F}(z)$ extreme properties can be a physical base to realize such kind of single receiving antenna scanning strategy.

The interference function $\overline{F}(z)$ has extreme value – maximum at $z = 0$, at last. One can verify this statement easily in the simple case of point random electric dipole source, with neglecting effect of absorption when interference function $F(z)$ has for small z the Taylor expansion

$$\frac{F(z)}{F(0)} = 1 - \frac{z^2}{2 z_{1/2}^2} + \cdots; \qquad \frac{R_0^2}{6 z_{1/2}^2} = \alpha - \frac{h^2}{R_0^2} \tag{67}$$

Here $F(0) = 4/R_0^2$ is maximum value of $F(z)$, with $R_0 = (R_\perp^2 + h^2)^{1/2}$ being equal to $R_\pm(z)$ at $z = 0$. The quantity $z_{1/2}$ is half width at half maximum (HWHM) of the Taylor expansion in Eq. (67). A constant α in the RHS of the second Eq.(67)

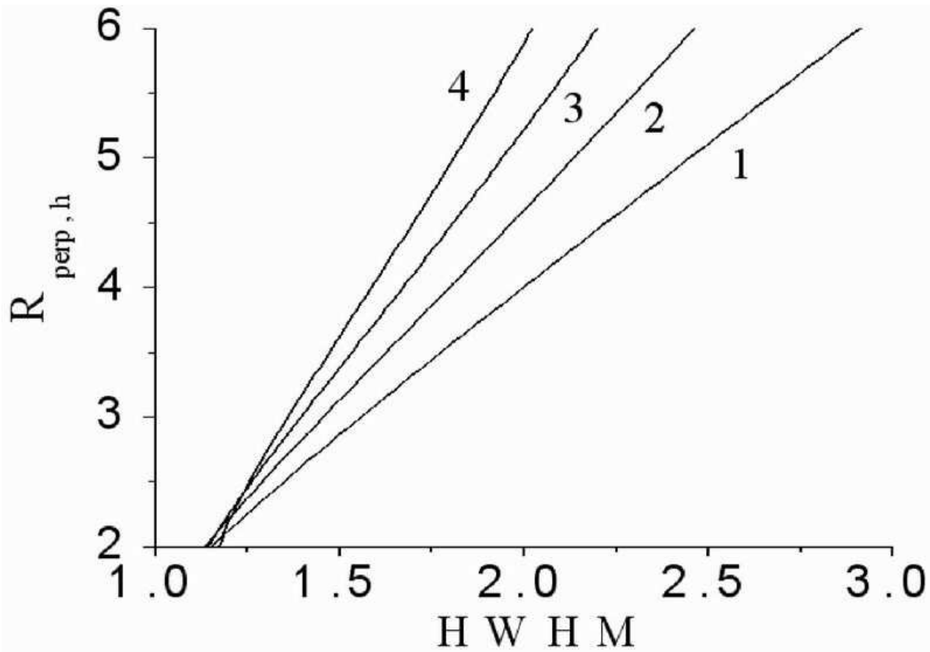

Figure 5. The normalized seeming depth $R_{\perp,h}$ versus HWHM of the dependences of normalized interference function calculated by Eq.(66) in the case of point source but accounting for an absorption $k_1'' h$ in the units of $n\lambda_1 / 8d_{sk}$ = $n \times 0.2182$: n = 1 (curve 2), 2 (3), and 3 (4). The reference curve 1 (no absorption) is the curve 1 in Fig.5(b).

is defined as $3\alpha = 1 + (\pi/3)^2$ for the half wavelength vibrator antenna.

Eq.(67) shows that HWHM of interference function $F(z)$ can be used to determine the seeming depth $R_\perp$ of random electric dipole point source. Because the HWHM is experimentally measured quantity we study next the interference function

$\overline{F}(z)$ extreme points and their HWHM in details, with taking into account effects of absorption and random electric dipole source extension.

Fig.4(a) presents calculated by Eq.(64) dependence of the normalized interference function $h^2 F(z_h)$ on normalized height $z_h = z / h$ of random electric dipole point source at a set of the point source normalized seeming depths $R_{\perp h} = R_{\perp} / h$ ranged from 1 up to 6 when the curves have only one peak and absorption is neglected. From this calculated curves we take HWHM and plot HWHM against the known normalized seeming depth $R_{\perp h}$ (Fig.4(b), curve 1). The curve 2 in the Fig.4(b) plots HWHW according to the Taylor expansion in Eq. (67) that is the quantity $(z_{1/2}) / h$. The Fig. 4 shows that HWHM of interference function growths monotonously with growing the random electric dipole point source seeming depth when effect of biological object absorption is neglected. The curve 2 in Fig.4(b) predicts substantially smaller value for HWHM at given $R_{\perp h}$ compared with the curve 1 in Fig.4(b) since the Taylor expansion dealing with only the peak top of the curves in the Fig.5(a). In Fig. 5 we repeat the curve 1 from Fig.4(b) and test the role of absorption in the case of point source. The absorption is measured in the units of $k_1'' h = n \lambda_1 / 8 d_{sk} = n \times 0.2182$ where n = 1, 2, and 3. The units of absorption measurement are chosen, with taking into account that for tuned vibrator-dipole $k_1' h = n\pi / 2$ and for the human head brain $\lambda_1 / d_{sk} = 7/4$, according to Introduction. Fig.5 shows that the more absorption is the less HWHM corresponds to the given point source seeming depth. In another words, the maximum peak of interference function $F(z)$ at $z = 0$ becomes narrower with growing absorption. Figs.6(a,b) depict HWHM of interference function against the normalized seeming depth $R_{\perp h}$ at a set of random electric dipole source normalized extensions $\Delta z / h$,

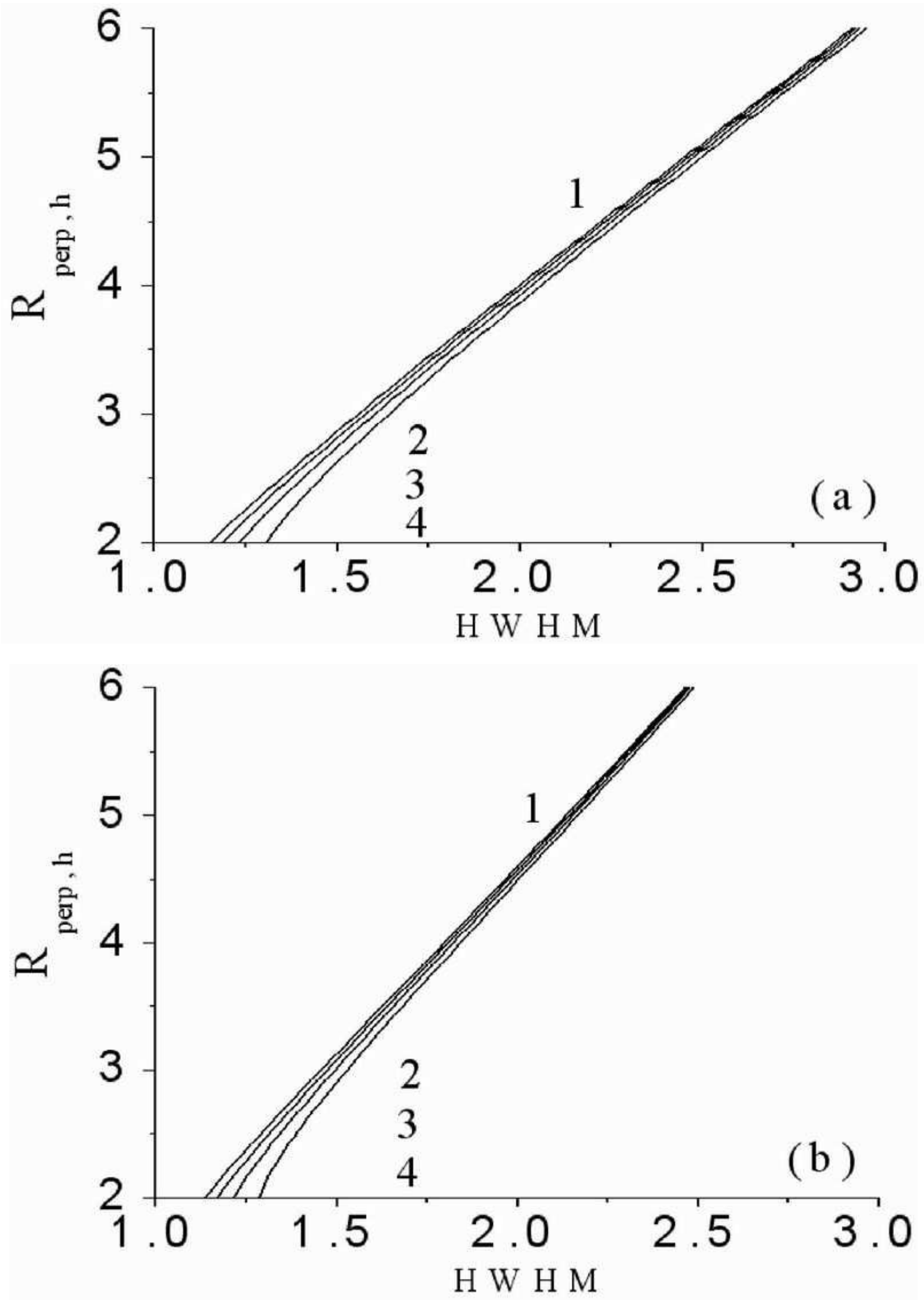

Figure 6. The normalized seeming depth $R_{\perp,h}$ versus HWHM of the dependences of normalized interference function calculated by Eq.(66) with neglecting of absorption (**a**) and with absorption accounting (**b**) in the case of point source (curve 1) and extended source $\Delta z / h$ = 1 (curve 2), 1.5 (3), and 2 (4). The absorption is equal to $k_1'' h = \lambda_1 / 8 d_{sk} = 0.2182$. The curves are enumerated by the figures in a manner that the lower is the curve the higher is its number.

with neglecting absorption, Fig.6(a), and accounting absorption, Fig.6(b). It is seen from the panels (a) and (b) of Fig.6 that the absorption leads only to an increase in the slope angle of a “bundle” of the curves with respect to the HWHM axis. From the other hand, comparison of Fig.5 and Figs.6 demonstrates that absorption and source extension make change in different manner the HWHM dependence on source centre seeming depth. As one can note, the maximum peak of interference function $\overline{F}(z)$ at $z = 0$ becomes wider with growing the source extension in both cases of neglecting and accounting absorption. In addition to noted the effects of absorption and source extension are more sensitive for relatively big and small source centre seeming depths, respectively. Fig.5 curves allow one as to determine the point source seeming depth at given absorption as Fig.6(b) curves permit one to get a source centre seeming depth, with knowing biological object absorption and the source extension. If in the last case the source extension is known approximately with some accuracy the source centre seeming depth is obtained also approximately with corresponding accuracy.

Having described Fig.4(a), we mentioned that the normalized interference function curves have only peak when source normalized seeming depth ranged from 1 up to 6 and absorption is neglected. Fig.7(a,b) shows that for smaller values of source normalized seeming depth the normalized interference function curves can have several peaks, with possibility for side peaks being not less the central peak (Fig.7, curve 4 has three equal peaks). The side peaks may be studied in manner similar to the central peak consideration. Nevertheless we will not study side peaks here.

Figs.4, 5, and 6 show that one can actually get the random electric dipole source centre inside the X_1, Y_1-plane of a single receiving linear wire antenna (Fig.3), via scanning this antenna along the Z_1- axis on the biological object boundary surface y_1 = 0 and defining the HWHM of antenna interference function $\overline{F}(z)$ maximum at z_1 = 0. As this takes place we obtain the source centre seeming depth

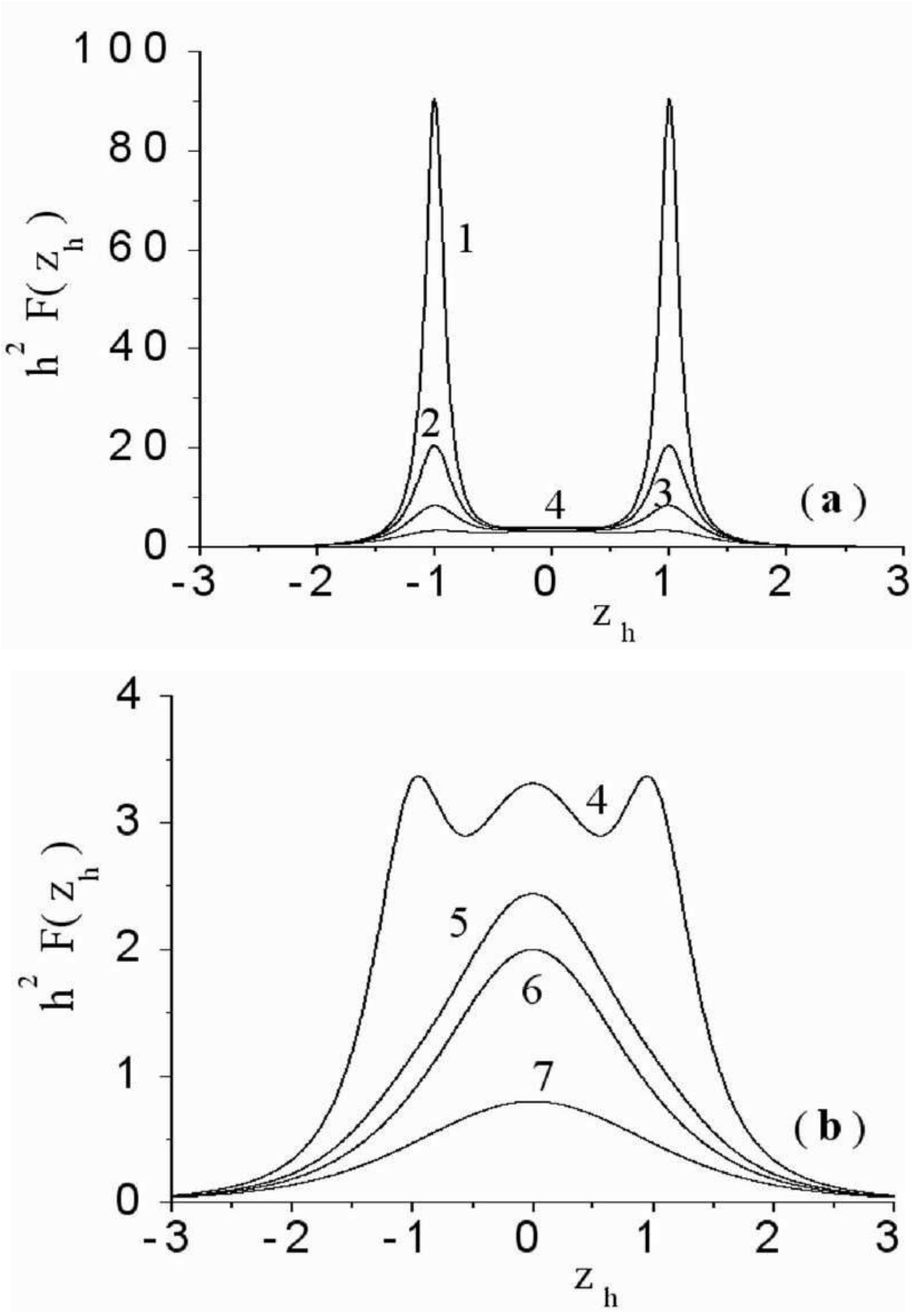

Figure 7. The normalized interference function versus normalized height of random electric dipole point source with neglecting object absorption at different relatively small values of the point source normalized seeming depths: (**a**) $R_{\perp,h}$ = 0.1 (curve 1), 0.2 (2), 0.3 (3), and; (**b**) $R_{\perp,h}$ = 0.455 (curve 4), 0.8 (5), 1 (6), and 2 (7).

$R_\perp$, with knowing the object absorption and source extension. Now we intend to define the real depth $R_{\perp 0}$ of the source centre in the centre of $2D$ position inside the X_1, Y_1-plane, via antenna scanning along the X_1-axis on biological object boundary surface.

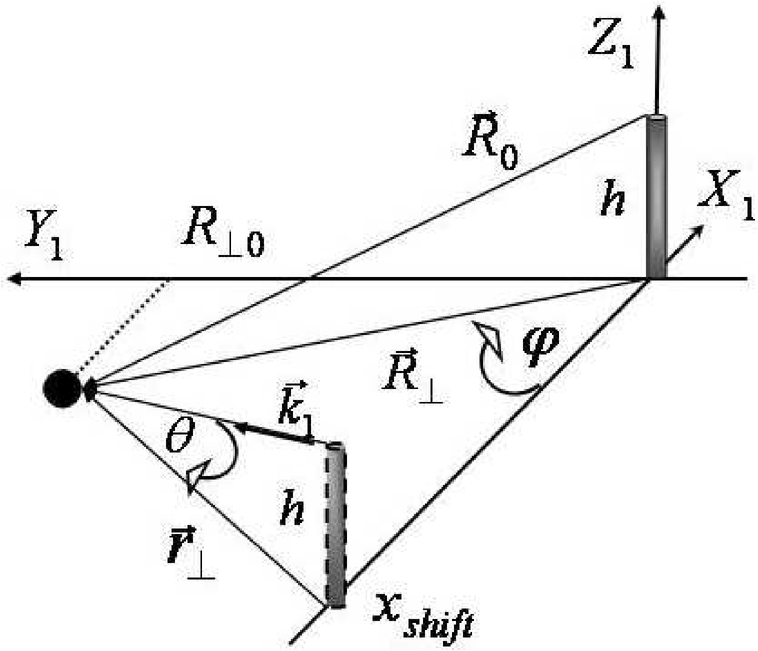

Figure 8. Schematic showing of the upper half space of the X_1, Y_1- plane with antenna shifted from the origin point to the point with the x_{shift} coordinate along the X_1 axis. The symbol (●) shows the centre of the random electric dipole extended source on the X_1, Y_1-plane. Vector $\vec{r}_\perp$ is a current value of the vector $\vec{R}_\perp$ when the antenna has the coordinate x_{shift}.

Fig.9 depicts schematically scanning the random electric dipole extended source with centre on the X_1, Y_1-plane by shifting the single receiving linear wire antenna along X_1-axis to position with coordinate $x_{shift} \le 0$. The interference function $\overline{F}(x_{shift})$ of the shifted single antenna is obtained from Eq.(66) by setting $z = 0$ and has a form

$$h^2 \overline{F}(x_{shift,h}) = 2A_0^2 \left[1 + \Gamma\left(\frac{k_1' h \Delta z_{,h}}{r_{0,h}} \right) \right] \quad (68)$$

with $A_0 = \exp(-k_1'' h\, r_{0,h}) / r_{0,h}$ where $r_{0,h} = (r_{\perp,h}^2 + 1)^{1/2}$. The quantity $r_{\perp,h}$ is function of shifted antenna position and given (Fig. 8) by relation

$$r_{\perp,h}^2 = R_{\perp 0,h}^2 + \left(x_{shift,h} - R_{\perp x,h}\right)^2 \tag{69}$$

where $R_{\perp x,h} = -R_{\perp,h} \cos\varphi \le 0$, $0 \le \varphi \le \pi/2$ is projection of vector $\vec{R}_{\perp,h}$ on the X_1-axis. All quantities in Eqs.(68) and (69), having dimension of length, are normalized to antenna half length h.

The interference function of the shifted single antenna in Eq.(68) has evidently maximum at $x_{shift} = R_{\perp x,h}$ where the scanning antenna is brought in nearest position to the random electric dipole extended source, with distant $r_{\perp,h}$ between source centre and antenna centre becoming equal to the real depth $R_{\perp 0,h}$ of the source centre. The two quantities $R_{\perp x,h}$ and $R_{\perp 0,h}$ are connected between them by relation $R_{\perp x,h}^2 + R_{\perp 0,h}^2 = R_{\perp,h}^2$. Therefore one can get the real depth of the source centre $R_{\perp 0,h}$, provided one knows from scanning experiment the antenna position $x_{shift} = R_{\perp x,h}$ where antenna interference function has maximum. We have also possibility to determine the real depth of the source centre by study the maximum peak of interference function in Eq.(68). The Taylor expansion for small values of $x_{shift,h} - R_{\perp x,h}$ gives a representation similar to one in Eq.(67) and written as

$$\frac{\overline{F}(x_{shift,h})}{\overline{F}(R_{\perp x,h})} = 1 - \frac{(x_{shift,h} - R_{\perp x,h})^2}{2\, x_{1/2}^2} + \cdots \tag{70}$$

Here $\overline{F}(R_{\perp x,h})$ is maximum value of $\overline{F}(x_{shift,h})$. The quantity $x_{1/2}$ is HWHM of the Taylor expansion in Eq. (70) defined by

$$\frac{1}{x_{1/2}^2} = \frac{2}{r_{00,h}^2}\left[1 + k_1'' h\; r_{00,h} + \Gamma_1\!\left(\frac{k_1' h\, \Delta z_{,h}}{r_{oo,h}}\right)\right] \tag{71}$$

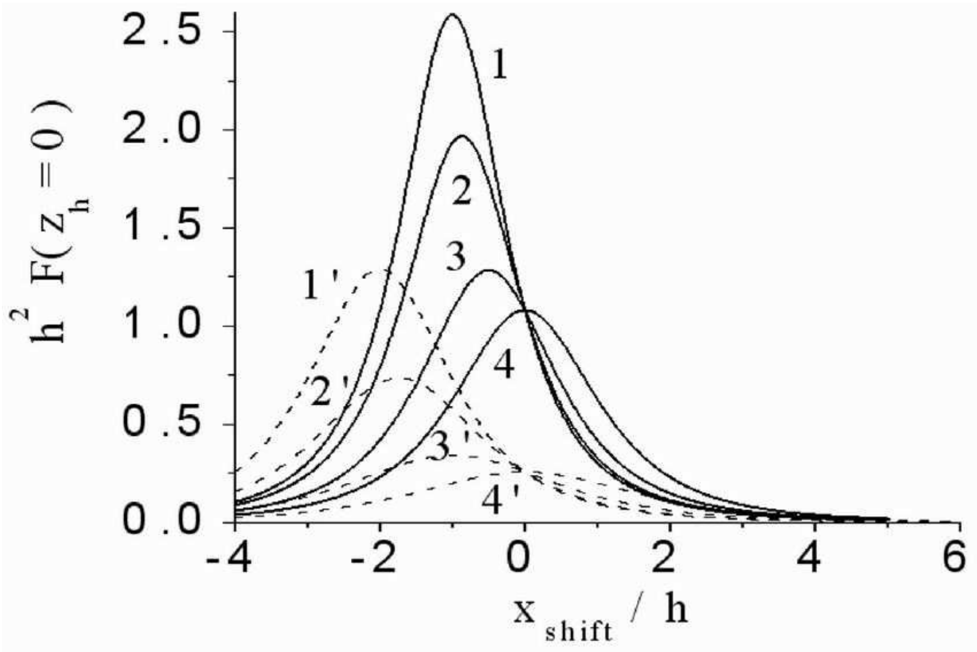

Figure 9. Dependence of the normalized interference function on a shift of the X_1-position of antenna from the origin point: solid lines numbered by the figures and dashed lines numbered by the same figures but with the stroke correspond to $R_{\perp,h}$ = 1, Δz_h = 0 and $R_{\perp,h}$ = 2, Δz_h = 2, respectively. Magnitude of the azimuth angle φ is equal to 1º (curves 1, 1'), 30º (2, 2'), 60º (3, 3'), and 90º (4, 4').

where quantity $r_{00,h} = (R^2_{\perp 0,h} + 1)^{1/2}$ and function $\Gamma_1(z)$ is defined according to

$$\Gamma_1(z) = \frac{1}{2} \frac{\cos z - \dfrac{\sin z}{z}}{1 + \dfrac{\sin z}{z}} \qquad (72)$$

Eq. (71) shows that the HWHM $x_{1/2}$ is related to the real depth of the source centre $R_{\perp 0,h}$ directly and becomes smaller with absorption growing. Bearing in mind an asymptotics $\Gamma_1(z) \approx -z^2/12$ as $z \to 0$, one can conclude also that the HWHM $x_{1/2}$ increases with taking into account a small extension of the source.

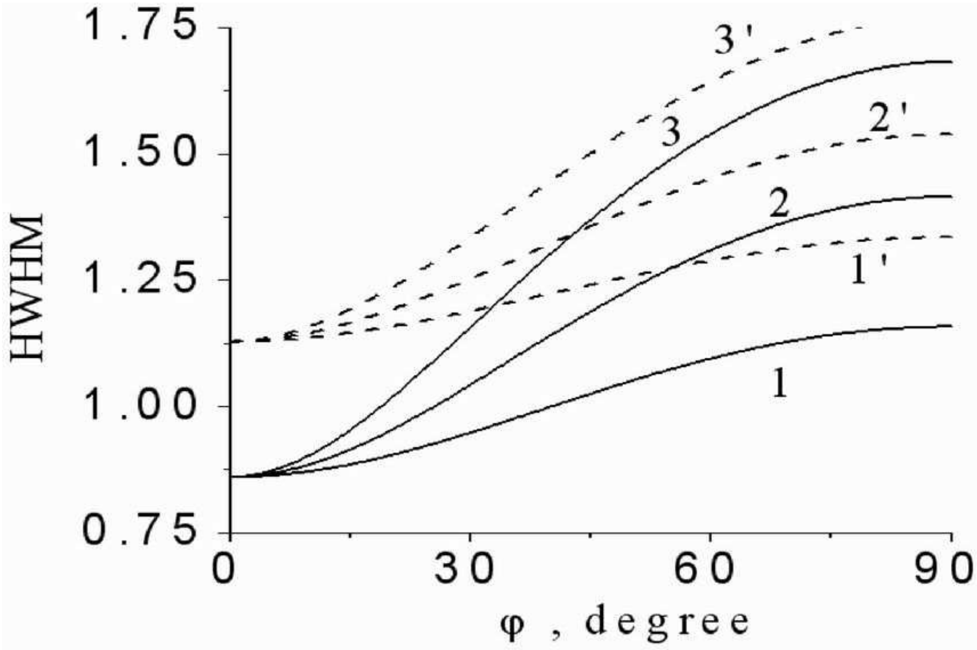

Figure 10. Dependence of HWHM of interference function Eq. (68) on the azimuth angle φ: solid lines numbered by the figures and dashed lines numbered by the same figures but with the stroke correspond to Δz_h = 0 and Δz_h = 2, respectively. Magnitude of $R_{\perp,h}$ is equal to $R_{\perp,h}$ = 1 (curves 1, 1'), 1.5 (2, 2'), and 2 (3, 3').

Fig.9 presents the normalized interference function from Eq.(68) versus to normalized shift $x_{shift,h}$ of scanning along X_1-axis antenna,. with no absorption taking into account. Fig.9 shows that normalized interference function from Eq.(68) shift towards negative X_1-axis direction with decreasing azimuth angle φ according to above position of the normalized interference function maximum. Meanwhile the normalized interference function becomes wider with growing the azimuth angle φ, that calls for growing the real depth of the source centre, and with growing the source extension also. Fig.10 presents the HWHM of normalized interference function from Eq.(68) versus to the varied azimuth angle φ at fixed the normalized seeming depth but for a set of source extension, with no absorption taking into account. Fig. 10 shows again that the normalized interference function becomes wider with growing the source extension. Fig. 11 generalizes content of Fig.10 on the subject to take into

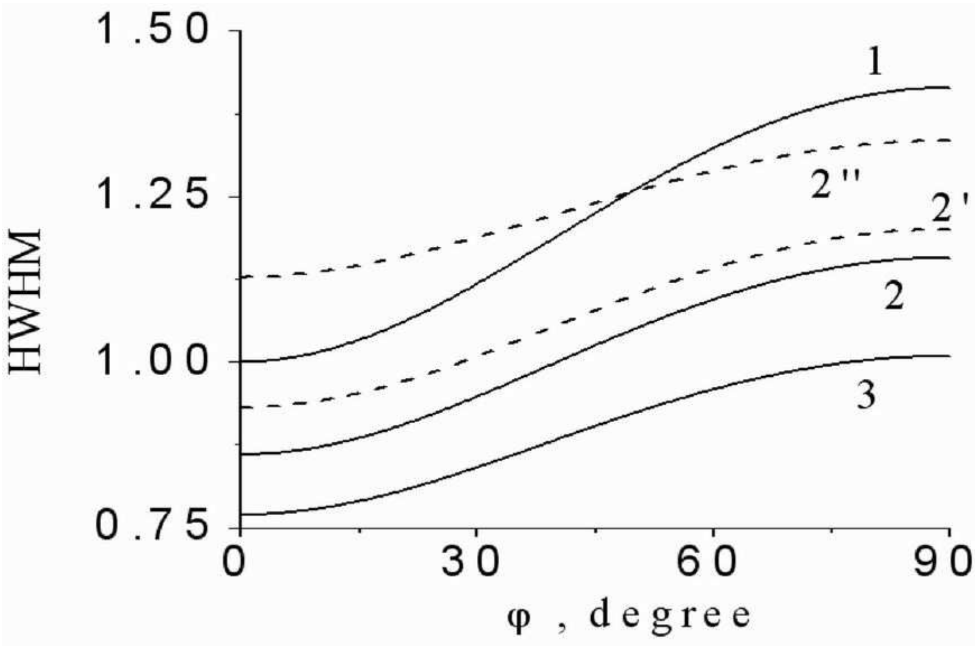

Figure 11. Dependence of HWHM of interference function Eq.(68) on azimuth angle φ at $R_{\perp,h}$ = 1: Δz_h = 0 (solid curves), $k_1''h$ = 0 (curve 1), $\lambda_1/8d_{sk}$ (2), and $2\lambda_1/8d_{sk}$ (3); $k_1''h$ = $\lambda_1/8d_{sk}$ (dashed curves), Δz_h = 1 (curve 2'), Δz_h = 2 (curve 2").

account the absorption, showing that the normalized interference function from Eq.(68) becomes narrower with growing absorption and wider with growing the source extension. Fig.12 presents the final dependence of normalized real depth of centre of random electric dipole extended source versus the HWHM of normalized interference function from Eq. (68), with taking into account absorption. The curves in Fig.12 show a competition between effects of absorption and source extension that make the HWHM smaller and bigger, respectively. As a result of such competition it is seen a crossing, in particular, of two curves (1 and 2") in Fig. 12.

Before going above to study the maximum peak of interference function in Eq.(68), we had mentioned that one can get the real depth of the source centre if one knows from scanning experiment the antenna position on X_1-axis where antenna interference function has maximum. In this case the three points' set consisting of antenna centre origin position, the just mentioned shifted antenna position and the source centre position inside the X_1, Y_1-plane (see Fig.8) form a rectangular triangle.

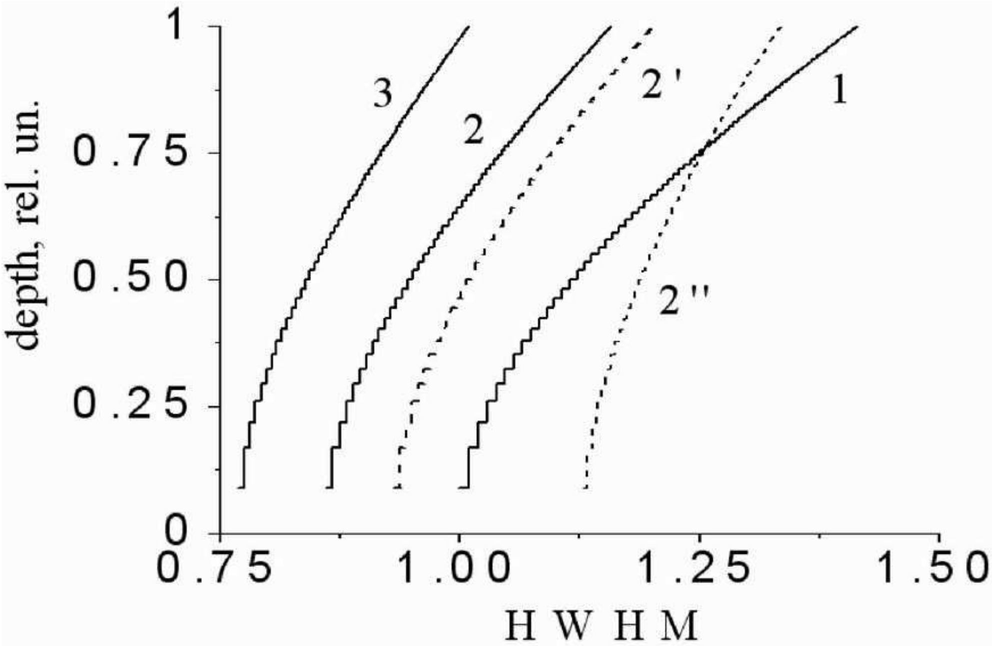

Figure 12. Dependence of the normalized depth of the source position on the HWHM (Fig.11) at $R_{\perp,h}$ = 1: Δz_h = 0 (solid curves), $k_1''h$ = 0 (curve 1), $\lambda_1/8d_{sk}$ (2), and $2\lambda_1/8d_{sk}$ (3); $k_1''h$ = $\lambda_1/8d_{sk}$ (dashed curves), Δz_h = 1 (curve 2'), Δz_h = 2 (curve 2'').

Let us note in order to generalize such kind forming a triangle that one can consider two positions of two single antennas 1 and 2 centers along X_1-axis of the X_1, Y_1, Z coordinate system (Fig. 13) when a many-side triangle is formed inside the X_1, Y_1-plane by centers of these two antennas and random electric dipole source centre projection on the X_1, Y_1-plane. The lengths $R_{1\perp}$ and $R_{2\perp}$ of formed triangle two sides can be determined separately via scanning the antennas 1 and 2 along Z_1- axis on biological object boundary surface $y_1 = 0$ and defining the HWHM of these antennas' interference functions $\bar{F}_1(z)$ and $\bar{F}_2(z)$ maximums at $z = 0$, in accordance with described study the interference function in Eq.(66). After that one can determine the real depth $R_{\perp 0}$ of random electric dipole source centre by resolving the triangle , with knowing its two mentioned sides as two seeming depths of source centre from two antennas centers and knowing distant between antennas. A symmetrical position of random electric dipole source relatively antennas, which transforms the above many-side triangle into isosceles one, is especially interesting for the case when effects of antennas' coupling are taking into account. This case of two coupled tuned receiving linear wire antennas' exciting by the random electric dipole source is our next task.

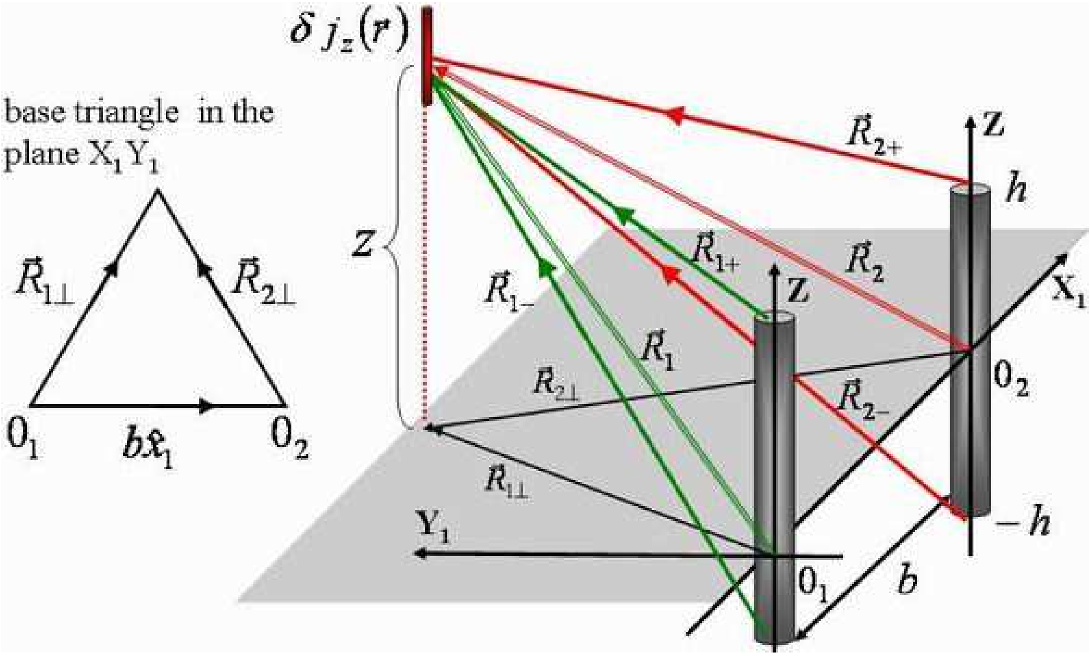

Figure 13. Schematic showing of two coupled antennas in thermal radiation field of local temperature inhomogeneity.

In the framework of using random electric dipole source model the z-component $\delta E_z^{\ 0}(z_1)$ of the incident random electric field along as coupled vibrator-dipole antenna 1 in Fig. 13 is given as along single vibrator-dipole antenna in Fig.3 by Eq.(58). Analogous expression for z- component $\delta E_z^{\ 0}(z_2)$ of the incident random electric field along coupled vibrator-dipole antenna 2 in Fig. 13 is obtained from Eq.(58) by replacing z_1 to z_2. The incident random electric fields $\delta E_z^{\ 0}(z_1)$ and $\delta E_z^{\ 0}(z_2)$ excite along two coupled antennas some current distributions with random amplitudes $\delta I_0^{(1)}$ and $\delta I_0^{(2)}$ given by Eqs.(41) and (42), respectively, with replacing $E_z{}^0(z_1)$ to $\delta E_z{}^0(z_1)$ and $E_z{}^0(z_2)$ to $\delta E_z{}^0(z_2)$ in the RHS integrands. Writing now equations similar to Eqs.(59) and (60) and supposing the tuned vibrator-dipoles to be of length equal to add whole number of half wavelengths lead us to generalization of Eq.(61) in the form

$$Z_1(1-a_{12}^2)\,\delta I_0^{(1)} = -(-1)^{(n-1)/2}\,\frac{k_1'}{i\omega\varepsilon'} \int dz' \left[\sum_{\mu=\pm} \frac{\exp\left(i k_1 R_{1\mu}(z')\right)}{R_{1\mu}(z')} + a_{12} \sum_{\mu=\pm} \frac{\exp\left(i k_1 R_{2\mu}(z')\right)}{R_{2\mu}(z')}\right] \delta I_z(z') \tag{73}$$

and

$$Z_1(1-a_{12}^2)\,\delta I_0^{(2)} = -(-1)^{(n-1)/2}\,\frac{k_1'}{i\omega\varepsilon'} \int dz' \left[\sum_{\mu=\pm} \frac{\exp\left(i k_1 R_{2\mu}(z')\right)}{R_{2\mu}(z')} + a_{12} \sum_{\mu=\pm} \frac{\exp\left(i k_1 R_{1\mu}(z')\right)}{R_{1\mu}(z')}\right] \delta I_z(z') \tag{74}$$

Here sums are taken over index μ two values $\pm$, with distances $R_{1\pm}(z')$ and $R_{2\pm}(z')$ being defined similar to the case of single antenna in Eq.(61) and related to antennas 1 and 2, respectively, in Fig. 13. According to Eqs.(73) and (74) the two spherical waves are propagated from a point z' of random electric dipole source towards receiving vibrator-dipole 1 ends as well as two another spherical waves are propagated from the same point of source towards receiving vibrator-dipole 2 ends. Bearing in mind the reciprocity between receiving and transmitting antennas one can say also that four spherical waves are propagated from vibrator- dipoles 1 and 2 ends towards the random electric dipole source and interfere on the source area. Eqs.(57) jointly with Eqs.(73) and (74) enables us to get for the fluctuations' spectral densities $\left\langle \left|\delta I_0^{(1,2)}\right|^2 \right\rangle$ of current distributions' along coupled receiving antennas 1 and 2 amplitudes caused by random electric dipole source thermal radiation the following equalities

$$\left(\frac{\omega\varepsilon'}{k_1'}\right)^2 \left|Z_1\right|^2 \left|1-a_{12}^2\right|^2 \left\langle\left|\delta I_0^{(q)}\right|^2\right\rangle = \frac{1}{4\pi^2}\,\omega\varepsilon''\Delta\Omega\ \delta\Theta\,\overline{F}^{(q)}(z) \tag{75}$$

which generalize Eq.(62) written for a single antenna. Functions $\overline{F}^{(q)}(z)$ in the RHS of these equalities, with indices $q=1{,}2$ related to coupled antennas 1 and 2, have form of integral averaging along random electric dipole source extension in Eq.(63), with integrands $F^{(q)}(z)$ being presented as

$$F^{(1)}(z) = F_1(z) + \left|a_{12}\right|^2 F_2(z) + 2\left|a_{12}\right| F_{12}(z) \tag{76}$$

and

$$F^{(2)}(z) = F_2(z) + \left|a_{12}\right|^2 F_1(z) + 2\left|a_{12}\right| F_{21}(z) \tag{77}$$

Functions $F_q(z)$ here coincides in physical sense with interference functions for point random electric dipole source exciting a single antenna 1 or 2 and are defined accordingly to Eq.(64) by

$$F_q(z) = A_{q+}^2(z) + A_{q-}^2(z) + 2A_{q+}(z)A_{q-}(z)\cos\left[k_1'\left(R_{q+}(z)-R_{q-}(z)\right)\right] \tag{78}$$

where $A_{q\pm}(z) = \exp\left[-k_1''R_{q\pm}(z)\right]/R_{q\pm}(z)$. While the functions in Eq.(78) we call the auto-interference functions of single antennas 1 and 2, the functions $F_{12}(z)$ and $F_{21}(z)$ need being called the cross-interference functions of coupled antennas because the last two functions take into account interference between a couple of spherical waves (see Fig. 13), one of which propagates from the point of random electric dipole source towards a receiving vibrator-dipole 1 end and another propagates to a receiving vibrator-dipole 2 end, as it is confirmed by equations

$$F_{12}(z) = \sum_{\mu,\nu=\pm} A_{2\mu}(z)A_{1\nu}(z)\cos\left[k_1'\left(R_{2\mu}(z)-R_{1\nu}(z)\right)+\alpha_{12}\right] \tag{78}$$

and $F_{21}(z) = F_{12}(z)\big|_{\alpha_{12}\to-\alpha_{12}}$ where α_{12} is phase of the coupling factor a_{12} defined by $a_{12} = \left|a_{12}\right|\exp(i\alpha_{12})$. The sum in the RHS of Eq.(78) is taken over both indices μ and ν two values $\pm$, with including four terms. The written equations for the cross- interference functions of antennas include an addition shift $\pm\alpha_{12}$ caused by antennas coupling side by side with phase shift $k_1'\left(R_{2\mu}(z)-R_{1\nu}(z)\right)$ equal to paths differences of two spherical waves in the biological object medium.

Consider dependence of antennas interference functions $F^{(q)}(z)$, defined in Eqs.(76) and (77) for the case of point random electric dipole source, on the antennas coupling factor a_{12}. We see this dependence in a simple form of scaling factors $\left|a_{12}\right|^2$ and $2\left|a_{12}\right|$ as well as in a complicate form of the addition phase shift in expressions for the cross-interference functions of coupled antennas. Nevertheless the complicate phase shift comes into $F_{12}(z)$ and $F_{21}(z)$ with opposite signs and hence transforms into a scale factor in the sum of these cross-interference functions

$$F_{12}(z)+F_{21}(z)=2\left(\cos\alpha_{12}\right)\sum_{\mu,\nu=\pm} A_{2\mu}(z)A_{1\nu}(z)\cos\left[k_1'\left(R_{2\mu}(z)-R_{1\nu}(z)\right)\right] \quad (79)$$

Summing next Eqs.(76) and (77), with accounting Eq.(79), gives

$$\begin{aligned} F^{(1)}(z)+F^{(2)}(z) &= \left(1+\left|a_{12}\right|^2\right)\left[F_1(z)+F_2(z)\right] \\ &+4\left|a_{12}\right|\left(\cos\alpha_{12}\right)\sum_{\mu,\nu=\pm} A_{2\mu}(z)A_{1\nu}(z)\cos\left[k_1'\left(R_{2\mu}(z)-R_{1\nu}(z)\right)\right] \end{aligned} \quad (80)$$

Summing at last the basic Eqs. (75) defined the fluctuations' spectral densities $\left\langle\left|\delta I_0^{(1,2)}\right|^2\right\rangle$ and reminding definition of functions $\bar{F}^{(q)}(z)$ give us

$$\left(\frac{\omega\varepsilon'}{k_1'}\right)^2\left|Z_1\right|^2\left|1-a_{12}^2\right|^2\left[\left\langle\left|\delta I_0^{(1)}\right|^2\right\rangle+\left\langle\left|\delta I_0^{(2)}\right|^2\right\rangle\right]=\frac{1}{4\pi^2}\,\omega\varepsilon''\Delta\Omega\;\delta\Theta\,\bar{F}^{(1+2)}(z) \quad (81)$$

with

$$\bar{F}^{(1+2)}(z)=\frac{1}{\Delta z}\int_{z-\Delta z/2}^{z+\Delta z/2} dz'\left[F^{(1)}(z')+F^{(2)}(z')\right] \quad (82)$$

The obtained three Eqs.(80), (81) and (82) show that sum of the fluctuations' spectral densities of current distributions' along coupled receiving antennas 1 and 2 amplitudes caused by random electric dipole source thermal radiation has dependence on antennas coupling factor a_{12} in a simple form of scaling factors $(1+|a_{12}|^2)|1-a_{12}^2|^{-2}$ and $4|a_{12}|(\cos\alpha_{12})|1-a_{12}^2|^{-2}$ only.

Although the RHS of Eq.(81) has a simple dependence on antennas coupling factor, the RHS of Eq. (80) leaves some difficult for study via complicate structure of four term sum, with each term describing interference of a waves' couple propagated from a point source towards different antennas' ends. Therefore we return to general Eq.(78) for the cross-interference function of coupled antennas and consider a mentioned above special case of symmetrical position of random electric dipole source relatively antennas(Fig. 13) when a many-side triangle, formed inside the X_1,Y_1-plane by centers of antennas and random electric dipole source centre projection on the X_1,Y_1-plane, becomes isosceles one. In this case of source symmetrical position we have relations $R_{1\mu}(z)=R_{2\mu}(z)$, with index $\mu=\pm$, that simplifies Eq.(78) immediately as

$$F_{12}(z)=F_{21}(z)=F(z)\cos\alpha_{12} \tag{83}$$

where $F_1(z)=F_2(z)=F(z)$ and $F(z)$ is given by Eq. (64). The final physically transparent result consists in equations

$$\left\langle\left|\delta I_0^{(1,)}\right|^2\right\rangle=\left\langle\left|\delta I_0^{(2,)}\right|^2\right\rangle=\frac{1}{|1-a_{12}|^2}\left\langle\left|\delta I_0\right|^2\right\rangle \tag{84}$$

where $\left\langle\left|\delta I_0\right|^2\right\rangle$ is presented in Eq.(62). Thus in the special case of source symmetrical position relatively antennas the both cross-interference functions of coupled antennas become equal to auto- interference function of single antenna accurate to scaling factor $\cos\alpha_{12}$ as well as the fluctuations' spectral densities of current distributions' along coupled receiving antennas 1 and 2 amplitudes become equal to fluctuations' spectral densities of current distributions' along single antennas accurate to scaling factor $|1-a_{12}|^{-2}$. Ultimately one can reduce the scanning problem of random electric dipole source via two coupled tuned receiving linear wire antennas to considered already such scanning problem via a single antenna, provided one is able to place the scanning source into symmetrical position relatively antennas and move the antennas 1 and 2 together, not separately, along Z-axes on biological boundary surface $y_1 = 0$ (Fig. 13).

Conclusions

Theory of electromagnetic wave multiple scattering by ensemble of dielectric and conductive bodies has been applied to study coupled receiving antennas. A basic exact system of Fredholm's second kind integral equations for electric currents excited inside antennas is derived and written in terms of the electric field tensor T-scattering operator of a single antenna, the electric field retarded Green tensor function of a background and the incident on antennas electric field. In this equations' system an antenna is a body with given complex dielectric permittivity on some frequency, and electric current excited inside the antenna means sum of volume conducting and displacement electric currents. The background medium can be inhomogeneous one with some complex dielectric permittivity. The kernels of derived integral equations' system are not singular for the case of no overlapping antennas, although the background electric field Green tensor function is singular in the origin. Such kind of three - dimensional singularity has been met really at study the wave integral equation for electric field inside single antenna in the homogeneous background, by verifying consistence this integral equation with boundary conditions on the antenna surface. As was demonstrated, one has to take into account two sorts of the homogeneous background electric field Green tensor function strong singularity: (i) electric field Green tensor function decomposition into a delta Dirac function term and principal part, and (ii) rule to bring out the second derivative outside the three-dimensional singular integral.

The derived integral equations' system for electric currents excited inside coupled antennas has been applied to study near field coherent effects caused by thermal microwave radiation incident electric field distribution along single or two coupled linear wire perfectly conducting receiving antennas in the form of thin vibrator- dipoles placed at heated biological object boundary surface and tuned to half wavelength in the object. After having been neglected wave interaction of antennas with biological object boundary surface and used asymptotic method of “big logarithm”, the T-scattering operator of single antenna in the form of tuned vibrator-dipole has become a separable wire T- scattering operator that lead to analytic evaluating the local total currents on two coupled receiving antennas and got a dimensionless antennas’ coupling factor. Effects of homogeneous and local inhomogeneous biological object temperature components on receiving antennas have been considered. Homogeneous temperature component was treated as source for equilibrium thermal radiation with standard form of incident on receiving antennas electric field spatial correlation function. This treatment led to a generalized Nyquist formula for currents’ fluctuations excited on coupled receiving vibrator-dipole antennas, with accounting the auto- correlation and cross- correlation functions of random electric field inside each antenna and on both antennas, respectively. More original results have been obtained at study effects of biological object temperature distribution local volume change in the model framework of random electric dipole source inside object absorption skin slab area, with dipole source being parallel to vibrator-dipole antennas parallel between themselves and placed on the object surface. In the case of single receiving vibrator-dipole antenna it was shown that two spherical waves are propagated from a point of random electric dipole source towards receiving vibrator-dipole ends. Bearing in mind the reciprocity between receiving and transmitting antennas one can say also that two spherical waves are propagated from vibrator- dipole ends towards the random electric dipole source and interfere on the source area. This physical interpretation led similarly with optics to single antenna auto-interference function. Extreme properties of single antenna auto-interference function depending on random electric dipole source extension and the source centre three-dimension position relatively receiving antenna on the biological object surface have been studied in details. It was shown, in particular, that the single antenna auto-interference function has maximum at source centre near antenna equatorial plane, with half width at half maximum (HWHM) becoming narrower and wider under biological object absorption and source extension growing, respectively. Ultimately a method of random electric dipole source scanning via single receiving vibrator-dipole antenna moving along the biological object surface was formulated. Two scanning most simple strategy was considered: (i) by defining via HWHM the two values of source centre seeming depth relatively two antenna positions with next evaluating the real depth of the source centre (many-side triangle strategy), and (ii) by getting a symmetrical position of source relatively two antenna positions that transforms the many-side triangle into a isosceles one (isosceles triangle strategy).

In the case of two coupled receiving vibrator-dipole antennas 1 and 2, as was shown two spherical waves are propagated from a point of random electric dipole source towards receiving vibrator-dipole 1 ends as well as two another spherical waves are propagated from the same point of source towards receiving vibrator-dipole 2 ends. Hence side by side with above single antennas 1 and 2 auto-interference functions, a cross-interference function of coupled antennas 1 and 2 has been introduced. The cross-interference function includes a complicate dependence on antennas coupling factor phase. Nevertheless, in the special case of source symmetrical position relatively antennas the cross-interference function of coupled antennas become equal to auto- interference function of single antenna, accurate to scaling factor equal to antennas coupling factor phase cosine. At the same time the fluctuations' spectral densities of current distributions' along coupled receiving antennas 1 and 2 amplitudes become equal to fluctuations' spectral densities of current distributions' along single antennas, accurate to scaling factor in the simple algebraic form of antennas coupling factor. Ultimately, the scanning problem of random electric dipole source via two coupled tuned receiving linear wire antennas has been reduced to such scanning problem via a single antenna, provided one is able to place the scanning source into symmetrical position relatively antennas and move the antennas 1 and 2 together, not separately, along biological object boundary surface.

Acknowledgments

This work was supported in part by Russian Foundation for Basic Research, Grant 09-02-00920-a, by the Russian Academy of Sciences project " Passive multichannel human radio- and acusto-thermotomography in near zone" and "Investigations of new types of photonic crystals for development of optoelectronic elements of infocommunicationic nets".

References

1. *Godik E.E., Gulyev Yu.V.* Functional imaging of the human body // IEEE Engineering in Medicine and Biology, 10, 21, 1991.

2. *Reznik A.N.* Quasistatic field of thermal radiation in theory of contact radiothermomatry // Izv. Vyssh. Uchebn. Zaved., Radiofiz, 34, 512, 1991.

3. *Reznik A.N., Vaks V.L.,Yurasova N.V.*Quasistationary field of thermal emission and near-field radiometry // Phys. Rev. E, 70, 056601-1, 2004.

4. *Gaikovich K.P., Reznik A.N., Vaks V.L.,Yurasova N.V.* New effect in near-field thermal emission // Phys. Rev. Lett, 88, 104302-1, 2002.

5. *Anzimirov V.L., Arkhipova N.A., Pasechnik V.I., Yanovich A.V.* Investigationtic of thermal excitation in human head brain core at functional tests by method of dynamic multichannel radiothermovision // Biomed. Radioelektron., 8, 22, 2000.

6. *Rytov S.M.* Theory of electric fluctuations and thermal radiation. Akad. Nauk SSSR, Moscow, 1953 [in Russian].

7. *Landau L.D., Lifshitz E.M.* Electrodynamics of continuous media. Gostechisdat, Moscow, 1957.

8. *Cilin V.P.,Ruchadze A.A.* Electromagnetic properties of plasma and plasma like bodies. Gostechisdat, Moscow, 1961.

9. *Levin M.L., Rytov S.M.* Theory of equilibrium thermal fluctuations in electrodidamics. Nauka, Moskow, 1967 [in Russian].

10. *Rytov S.M., Kravtsov Yu.A., Tatarskii V.I.* Principles of statistical radiophysics.Vol. 3. Wave propagation through random media. Springer, Berlin, 1989.

11. *Honl H., Maue W., Westpfahl K.* Theorie der beugung. Springer, Berlin, 1966.1961.

12. *Goldberger M.L., Watson K.M.* Collision theory. Wiley, New York, 1964.

13. *Newton R.G.,* Scattering theory of waves and particles. McGraw-Hill, New York, 1966.

14. *Frish V.* Probabilistic methods in applied mathematics. Vol.1, edited by Bharucha-Reid. Academic, New York, 1968.

15. *Barabanenkov Yu.N.* Multiple scattering of waves by ensemble of particles and the radiative transport// Sov. Phys. Uspekhi, 18, 673, 1975.

16. *Tsang L., Kong J.A.* Multiple scattering of electromagnetic waves by random distributions of discrete scatterers with coherent potential and quantum mechanical formalism// J. Appl. Phys., 51, 3465, 1980.

17. *Gulyaev Yu.V., Barabanenkov Yu.N., Barabanenkov M.Yu., Nikitov S.A.* Optical theorem for electromagnetic field scattering by dielectric structures and energy emission from the evanescent wave //Phys. Rev.E, 72, 1, 2005.

18. *Leontovich M.A, Levin M.L.* To theory of oscillation excitation in vibrator antennas// Journ. Thechn. Phys. 14,481, 1944.

19. *Hallen E.*Theoretical investigation into the transmission and receiving qualities of antennae// Nova Acta Regiae Soc. Sci. Upsaliensis, Ser.IV, No. 4, 1-44, 1938.

20. *Pocklington H.C.* Electrical oscillation in wire// Cambridge Philos. Soc. Proc., 9, 324, 1897.

21. *King R, Harrison C.W.* Mutual and self- impedance for coupled antennas// J. Appl. Phys.15, 481, 1944.

22. *King R.* The theory o f N coupled parallel Antennas// J. Appl. Phys. 21, 94, 1950.

23. *Markov G.T.* Approximate calculation of mutual impedance for antennas// Radiotechnik. 3, 36, 1948.

24. *Brillouin L.* The Radiation Resistance of Resonant Transmission Line //Radioelectricite Sommaire , p.147, April, 1922.

25. *Pistolkors A.A.* The radiation resistance of antennas and antenna combinations// Proc. IRE, 17, 526,1929.

26. *Bechmann R.* On the calculation of radiation resistance of antennas and antenna combination// Proc. IRE, 19, 461,1931.

27. *Stratton J.A.* Electromagnetic theory. McGray-Hill, New York, 1941.

28. *Foldy L.L.* The multiple scattering of waves // Phys. Rev. 67, 107, 1945.

29. *Lax M.* MultiSpringerple scattering of waves// Rev. Mod. Phys. 23, 287, 1951.

30. *Vasilyev E.N.* Exitation of rotation bodies. Radio and Communiction, Moscow, 1987.

31. *Van Bladen J.* Some remarks on Green’s dyadic for infinite space// IRE Trans. Ant. And Prpop. AP-9, 563, 1961.

32. *V.M. Finkelberg.* Dielectric permittivity of mixtures// Journ. Techn.Phys. 34, 509, 1964.

33. *Gelfand I.M., Shilov G.E.* Gwnwlalized functions. Vol. 1: poreties and operators. Academic Press, Boston, 1964.

34. *Mallet P., Guerin C.A., Sentenas A.* Maxwell-Garnet mixing rule in the presence of multiple scattering: Derivation and accuracy// Phys. Rev. b, 72, 014205, 2005.

35. *Mikhlin S.G., Prossdorf S.* Singular integral operators. Springer, 1987.

36 *Born M., E. Wolf E.* Principles of optics. Pergamon Press, New York , 1964.

37. *Balanis C.A.* Antenna theory (analysis and design). A John Wiley&Sons, New Jersey, 2005.

38. *Korn G.A., Korn T.M.* Mathematical handbook (for scientists and engeneers). McGray-Hill, New York, 1961.

39. *Nyquist H.* Thermal agitation of conductors // Phys. Rev. 29, 614, 1927.